ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-30

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Applications of derivatives **Class: XII**

Topic: Increasing and Decreasing functions **Date: 08.08.2020**

➤ Increasing and Decreasing Functions

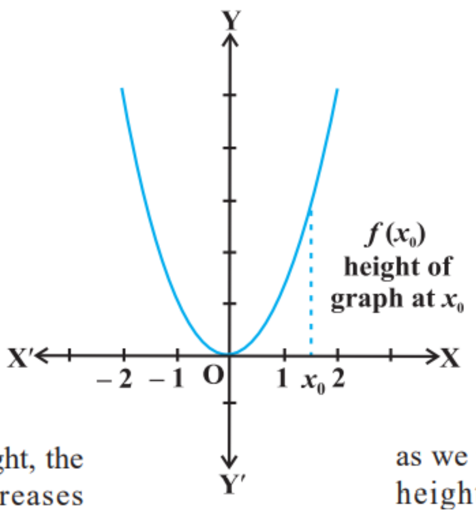
In this section, we will use differentiation to find out whether a function is increasing or decreasing or none.

Consider the function f given by $f(x) = x^2$, $x \in \mathbf{R}$. The graph of this function is a parabola as given in Fig

Values left to origin

x	$f(x) = x^2$
-2	4
$-\frac{3}{2}$	$\frac{9}{4}$
-1	1
$-\frac{1}{2}$	$\frac{1}{4}$
0	0

as we move from left to right, the height of the graph decreases



Values right to origin

x	$f(x) = x^2$
0	0
$\frac{1}{2}$	$\frac{1}{4}$
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4

as we move from left to right, the height of the graph increases

First consider the graph (Fig) to the right of the origin. Observe that as we move from left to right along the graph, the height of the graph continuously increases. For this reason, the function is said to be increasing for the real numbers $x > 0$.

Now consider the graph to the left of the origin and observe here that as we move from left to right along the graph, the height of the graph continuously decreases. Consequently, the function is said to be decreasing for the real numbers $x < 0$.

We shall now give the following analytical definitions for a function which is increasing or decreasing on an interval.

Definition 1 Let I be an open interval contained in the domain of a real valued function f. Then f is said to be

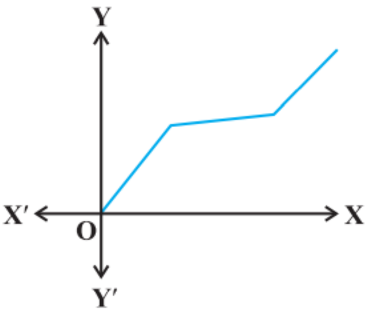
(i) increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in$ I.

(ii) strictly increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ I.

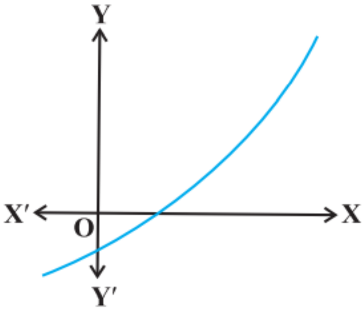
(iii) decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in$ I.

(iv) strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ I.

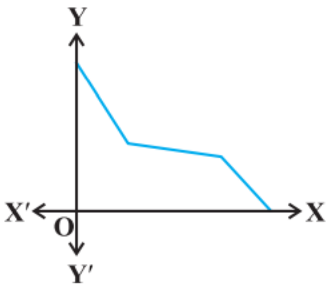
For graphical representation of such functions see Fig



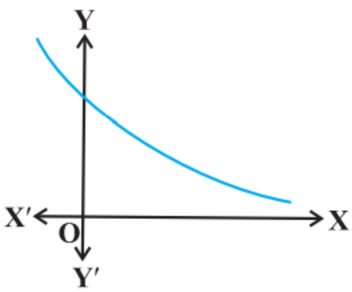
Increasing function
(i)



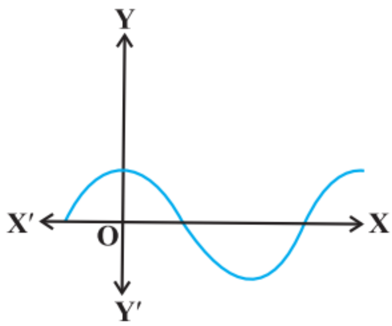
Strictly Increasing function
(ii)



Decreasing function
(iii)



Strictly Decreasing function
(iv)



Neither Increasing nor Decreasing function
(v)

We shall now define when a function is increasing or decreasing at a point.

Definition 2 Let x_0 be a point in the domain of definition of a real valued function f. Then f is said to be increasing, strictly increasing, decreasing or strictly decreasing at x_0 if there exists an open interval I containing x_0 such that f is increasing, strictly increasing, decreasing or strictly decreasing, respectively, in I.

Let us clarify this definition for the case of increasing function.

A function f is said to be increasing at x_0 if there exists an interval I = $(x_0 - h, x_0 + h)$, $h > 0$ such that for $x_1, x_2 \in$ I

$$x_1 < x_2 \text{ in I} \Rightarrow f(x_1) \le f(x_2)$$

Similarly, the other cases can be clarified.

Theorem 1 Let f be continuous on $[a, b]$ and differentiable on the open interval (a,b). Then

(a) f is increasing in $[a,b]$ if $f'(x) > 0$ for each $x \in (a, b)$

(b) f is decreasing in $[a,b]$ if $f'(x) < 0$ for each $x \in (a, b)$

(c) f is a constant function in $[a,b]$ if $f'(x) = 0$ for each $x \in (a, b)$

Remarks

(i) f is strictly increasing in (a, b) if $f'(x) > 0$ for each $x \in (a, b)$

(ii) f is strictly decreasing in (a, b) if $f'(x) < 0$ for each $x \in (a, b)$

(iii) A function will be increasing (decreasing) in **R** if it is so in every interval of **R**.

Example Show that the function f given by

$$f(x) = x^3 - 3x^2 + 4x,\ x \in \mathbf{R}$$

is strictly increasing on **R**.

Solution Note that

$$\begin{aligned} f'(x) &= 3x^2 - 6x + 4 \\ &= 3(x^2 - 2x + 1) + 1 \\ &= 3(x - 1)^2 + 1 > 0, \text{ in every interval of } \mathbf{R} \end{aligned}$$

Therefore, the function f is strictly increasing on **R**.

Example Prove that the function given by $f(x) = \cos x$ is

(a) strictly decreasing in $(0, \pi)$

(b) strictly increasing in $(\pi, 2\pi)$, and

(c) neither increasing nor decreasing in $(0, 2\pi)$.

Solution Note that $f'(x) = -\sin x$

(a) Since for each $x \in (0, \pi)$, $\sin x > 0$, we have $f'(x) < 0$ and so f is strictly decreasing in $(0, \pi)$.

(b) Since for each $x \in (\pi, 2\pi)$, $\sin x < 0$, we have $f'(x) > 0$ and so f is strictly increasing in $(\pi, 2\pi)$.

(c) Clearly by (a) and (b) above, f is neither increasing nor decreasing in $(0, 2\pi)$.

Prepared by : -

Mr. Sukumar Mandal (SkM)