ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-9

SUBJECT – STATISTICS

First term

Chapter: Central tendency **Class: XI**

Topic: Mode **Date: 20.07.2020**

CENTRAL TENDENCY

PART 5

Mode is the observation of a variable which has maximum frequency or frequency density, according as the variable is discrete or continuous.

Mode of variable x is denoted by $\breve{x}$

Case 1: In case of raw or ungrouped data

$x_1, x_2, \ldots \ldots \ldots. , x_n$

$\breve{x} = x_{i\,,} \forall\, i = 1(1)n$

Case 2: In case of discrete grouped data

Obsevation: $x_1 \quad x_2 \quad \ldots \ldots \ldots. \quad x_n$

Frequency: $f_1 \quad f_2 \quad \ldots \ldots \ldots. \quad f_n$

$\breve{x} = x_{\alpha\,,}$ iff, $\quad f_i > f_\alpha \;\; \forall\, i \neq \alpha$

Case 3: In case of continuous grouped data

Given class boundaries and frequencies of the respective class first we need to find the class which contains maximum frequency density. That class is known as modal class.

Define, f_0 : Frequency of the modal class

f_1 : Frequency of the class next to the modal class

f_{-1} : Frequency of the the class prior to modal class

c_0 : width of the modal class

c_1 : width of the class next to the modal class

c_{-1} : width of the the class prior to modal class

The formula of mode is defined as $\breve{x} = x_l + \frac{\frac{f_0}{c_0} - \frac{f_{-1}}{c_{-1}}}{2\frac{f_0}{c_0} - \frac{f_{-1}}{c_{-1}} - \frac{f_1}{c_1}} c_0$

Property of mode:

Change of base or origin and scale

If $y_i = a + b\, x_i$, $\forall\, i = 1(1)n$

then $\breve{y} = a + b\, \breve{x}$

Proof: Define the width of ith class of variable y is $c_i^{/}$

And the width of ith class of variable x is c_i

Now, $c_i^{/} = y_{ui} - y_{li} = a + bx_{ui} - a - bx_{li} = b\,(x_{ui} - x_{li}) = bc_i$

So by definition

$$\breve{y} = y_l + \frac{\frac{f_0}{c_0^{/}} - \frac{f_{-1}}{c_{-1}^{/}}}{2\frac{f_0}{c_0^{/}} - \frac{f_{-1}}{c_{-1}^{/}} - \frac{f_1}{c_1^{/}}} c_0^{/}$$

$$= a + bx_l + \frac{\frac{f_0}{bc_0} - \frac{f_{-1}}{bc_{-1}}}{2\frac{f_0}{bc_0} - \frac{f_{-1}}{bc_{-1}} - \frac{f_1}{bc_1}} bc_0$$

$$= a + b\, \breve{x}$$

Requirements of an ideal central measure:

An ideal measure should posses the following properties.

1) rigidly defined

2) based on all observations

3) capable of simple interpretation

4) easy to compute

5) readily amenable to algebraic treatment

6) more or less stable in case of sampling fluctuation.

Comparison between the measures of central tendency

The mean and mod are rigidly defined. The median is also rigidly defined , except when there is an even number of observations.

In finding each of this measures, all the observations are taken into consideration. However, only mean is directly based on all values and its value changes even if a single observation is altered. On the other hand, median and mode may remain unchanged even after alteration of several observations.

The significance of all these measures is quite easily comprehensible.

In general, the labour involved in computation of all these measures is almost the same. But in practise, the exact determination of mode of a continuous variable is impossible because practically we never get an ideal distribution.

The mean has several properties of virtue of them it can be readily manipulated in varied situations. But the median and mode do not posses such desirable properties.

Among three measures, mean is generally found to be least affected by sampling fluctuation. However, in this respect, the median or mode may be better than the mean in some specific situation.

Thus, it is evident that, in general, mean is the best measure of central tendency. But there are situations where mean cannot be or should not be used. In the case of root frequency distribution if at least one of the terminal classes be open we cannot use mean. Also in case of presence of an outlier median or mode seems to be the better central measure.

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