ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL - 3 (PART - I)

Subject : PHYSICS

CLASS : XII

Date : 18.5.20

Chapter : Electrostatics

Topic : Capacitor

A : Important formulae concepts, diagrams and explanation :

1. i) Capacitance, $C = q/V$
   ii) Capacitance of a spherical conductor $C = 4\pi\epsilon_0 r$

2. i) Capacitance of a parallel plate capacitor with air as dielectric is given by $C_0 = \frac{\epsilon_0 A}{d}$
   ii) Capacitance of a parallel plate capacitor with dielectric of dielectric constant K, and thickness 't' is given by

$$C = \frac{C_0}{1 - \frac{t}{d}\left(1 - \frac{1}{K}\right)}$$. If $t = d$, then $C = KC_0$

3. i) Equivalent Capacitance when Capacitors are in series is given by, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$
   ii) Equivalent Capacitance when Capacitors are in parallel is given by, $C_{eq} = C_1 + C_2 + C_3 + ...$

4. i) Energy sotred in a capacitor, $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{QV}{2}$
   ii) Common potential, $V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$
   iii) Energy loss on sharing charges $= U_1 - U_2 = \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}$

5. **Energy stored per unit volume or energy density between the plates :**

Energy stored per unit volume, $u = \frac{U}{ad} = \frac{\sigma^2}{2k\epsilon_0}$

This is called the **energy density in the electric field** of the capacitor.

Now, the electric field is uniform, except at the ends, inside a parallel plate capacitor, provided the plate area is very large compared to the separation between the plates.

From Gauss' theorem, we already know that the uniform electric field between the two plates of a parallel plate capacitor, neglecting end effects, is

$E = \frac{\sigma}{k\epsilon_0}$; then $\sigma = k\epsilon_0 E$

So, the energy density between the two plates is, $u = \frac{(k\epsilon_0 E)^2}{2k\epsilon_0} = \frac{1}{2}k\epsilon_0 E^2$ ....(1)

For vacuum or air, k = 1. Then equation (1) becomes $u = \frac{1}{2}\epsilon_0 E^2$ ....(2)

B : Solved numerical Problems

1. Find the capacitance of an isolated sphere of radius 15 cm

   Solution :

   Here r = 15 cm = 15 x 10^{-2} m

   Capacitance of an isolated sphere is given by

   $$C = (4\pi \in_0)r = \left(\frac{1}{9\times10^9}\right)(15 \times 10^{-2})$$

   $$= 1.667 \times 10^{-11} F$$

2. 75% of the distance d between the parallel plates of a capacitor is filled with a material of dielectric constant k. Find the change in capacitance if original capacitance was C_0.

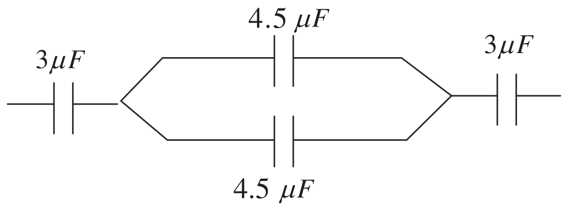
   Solution : Using the relation,

   $$C = \frac{C_0}{1 - \frac{t}{d}\left(1 - \frac{1}{K}\right)}, \text{ we get}$$

   $$C = \frac{C_0}{1 - \frac{75d}{100d}\left(1 - \frac{1}{K}\right)} \qquad (\because t = 75\% \text{ of } d)$$

   $$= \frac{4KC_0}{4K - 3K + 3} = \frac{4K}{K+3}C_0$$

3. Calculate the equivalent capacitance in the following circuit.

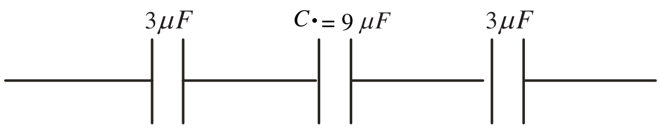


   Solution :

   Equivalent capacitance of 4.5 μF capacitors is given by,

   $$C' = 4.5\ \mu F + 4.5\ \mu F = 9\ \mu F$$

   Now $C' = 9\mu F$ is in series with other two capacitors as shown in figure below :

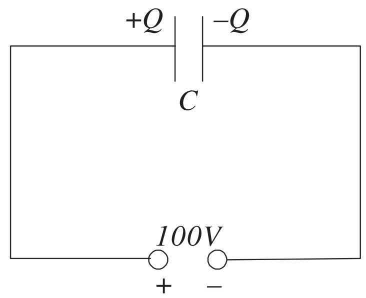


   Therefore, ***equivalent capacitance*** of the circuit is given by,

   $$\frac{1}{C} = \frac{1}{3} + \frac{1}{9} + \frac{1}{3} = \frac{3+1+3}{9} = \frac{7}{9}$$

   or $$C = \frac{9}{7} = 1.29\mu F$$

4. A 900 pF capacitor is charged by 100V battery as shown in figure. How much energy is stored by the capacitor and what is the charge on the capacitor ?

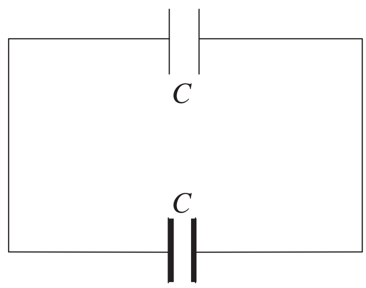


Solution : $C = 900\ pF = 900 \times 10^{-12}$
$= 9 \times 10^{-10} F$
$V = 100V$

Energy stores $U_1 = \frac{1}{2}CV^2 = \frac{1}{2} \; 9 \times 10^{-10} \times 10^4$
$= 4.5 \times 10^{-6} J.$

Charge on capacitor,
$Q = CV = 9 \times 10^{-10} \times 100 = 9 \times 10^{-8} C.$

5. From the Q. No. 4, now the capacitor is disconnected from the battery and connected to another 900 pF capacitor as shown in figure. What is the energy stored by the system?



Ans. When two capacitors are connected to each other as shown in the figure, they have common potential V'. Now charge on each capacitor, $Q' = CV'$.

According to the law of conservation of charge, $Q' + Q' = Q$ or $2Q' = Q$

or $2CV' = CV$ or $V' = \frac{V}{2} = \frac{100}{2} = 50V$

$\therefore$ Energy stored $U_2 = \frac{1}{2}C_1V'^2 + \frac{1}{2}C_2V'^2$ i.e. $U_2 = CV'^2$ $(\therefore C_1 = C_2 = C)$

$= 9 \times 10^{-10} \times 2500 = 2.25 \times 10^{-6} J.$

6. In a Van de Graaff generator, the shell electrode is at $25 \times 10^5\ V$. The dielectric strength of the gas surrounding the electrode is $5 \times 10^7\ V\ m^{-1}$. Calculate the minimum radius of the spherical shell.

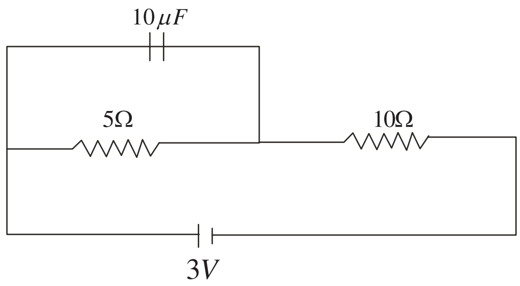
Solution : Electric potential of the charged shell $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$ .... (i)

Electric field of the charged shell $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ .... (ii)

As $E = \frac{V}{r}$ or $r = \frac{V}{E} = \frac{25 \times 10^5}{5 \times 10^7} = 5 \times 10^{-2}\ m = 5\ cm$

C : Solution of previous years questions

1. a) What will be the charge on the capacitor in the circuit given below?



b) Find the energy stored in the capacitor. (2018)

Ans. : a) Current in the circuit $I = \frac{V}{R} + \frac{3}{5+10} = \frac{1}{5} A$

Potential across 5Ω resistance = $\frac{1}{5} \times 5 = 1$ V.

Charge on the capacitor $Q = CV = (10 \times 10^{-6} \times 1)\ C = 10\,\mu C$

b) Energy stored, $U = \frac{Q^2}{2C} = \frac{100 \times 10^{-12}}{2 \times 10 \times 10^{-6}} = 5 \times 10^{-6}\ J$

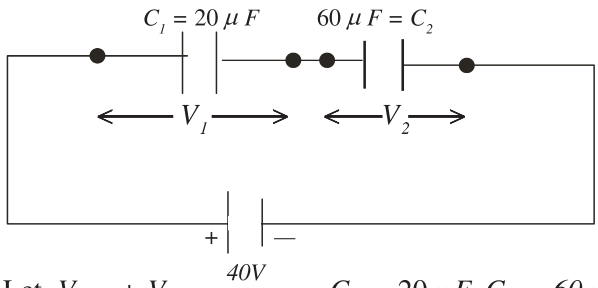
2. (a) On what factors does the capacitance of a capacitor depend?

(b) Two capacitors of capacitances 20μF and 60μF are connected in series. If the potential difference between the two ends of the combination is 40 volt, calculate the terminal potential difference of each capacitor. (2019)

Ans : (a) The value of capacitance of a conductor depends on the following factors —

surface area and shape of the conductor,
nature of the surrounding medium and
presence of other conductors (especially earthed ones)

(b)



Let, $V = + V_2$, $C_1 = 20\,\mu F$, $C_2 = 60\,\mu F$ in series,

$$\therefore\ C = \frac{C_1 C_2}{C_1 + C_2} = \left(\frac{20 \times 60}{80}\right) \mu F = 15 \mu F$$

Same charge (q) will flow through them,

$$q = CV = (15 \times 40)\ \mu C = 600\,\mu C$$

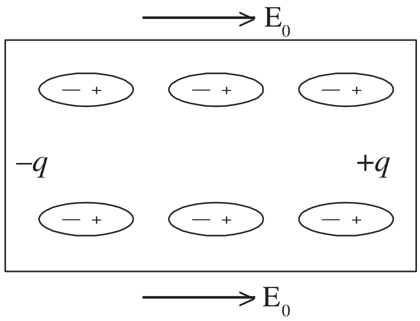
Now, $V_1 = \frac{q}{C_1} = \frac{600}{20} = 30V$

$$\therefore\ V_2 = \frac{q}{C_2} = \frac{600}{60} = 10V$$

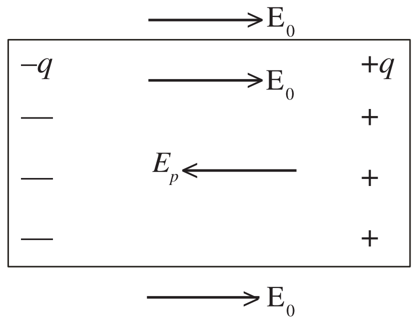
3. a) Define dielectric polarisation.

b) Deduce an espression for the potential energy stored in a parallel plate capacitor (2017)

Ans. a) Dielectric polarization is the term given to describe the behaviour of a material (non-polar dielectric) when an external electric field is applied on it, then



(i)



(ii)

[E_P = electric field inside the dielectric
E_0 = applied electric field]

$-q$ and $+q$ on the two surfaces of the dielectric slab are called induced charges. The induced charges set up an electric field E_P inside the dielectric. It is called electric field due to polarisation.

$\therefore \quad \vec{E} \quad \vec{E_0} \quad \vec{E_p}$

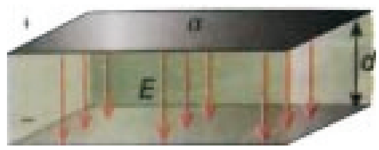
$\therefore$ Dielectric constant, $K = \frac{E_0}{E} = \frac{E_0}{E_0 - E_P}$

b) **Energy stored in a charged parallel plate capacitor :**

Let us consider a charged paralll plate capacitor. Here

a = area of each plate,

d = separation between the plates,



k = dielectric constant of the material between the plates,

σ = surface density of charge on each plate

So, volume between the plates = ad

and amount of charge on each plate $(Q) = \sigma a$.

The capacitance of this parallel plate capacitor,

$$C = \frac{k\epsilon_0 a}{d}$$

where ϵ_0 = permittivity of air or vacuum

Therefore, the energy stored in this charged capacitor,

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\left(\sigma^2 a^2\right)\frac{d}{k\epsilon_0 a} = \frac{\sigma^2 ad}{2k\epsilon_0}$$

The unit of U is joule (J). This energy is stored in the electric field between the plates of the capacitor.

[In CGS system, the expression for U is obtained by replacing ϵ_0 by $\frac{1}{4\pi}$.

So, $U = \frac{2\pi\sigma^2 ad}{k}$; its unit is erg.]

4. The equivalent capacitances of the parallel and the series combinations of two capacitors are $5\,\mu F$ and $1.2\,\mu F$, respectively. Calculate the capacitances of each capacitor. (2012)

Solution : Let the capacitances of the two capacitors be $C_1\,\mu F$ and $C_2\,\mu F$. According to the question,

$C_1 + C_2 = 5$

and $\dfrac{C_1C_2}{C_1+C_2} = 1.2$

or, $C_1C_2 = 1.2 \times 5 = 6$

or, $C_1(5 - C_1) - 6 = 0$ [with the help of equation (i)]

or, $C_1^2 - 5C_1 + 6 = 0$ or, $(C_1 - 3)(C_1 - 2) = 0$

$\therefore$ $C_1 = 3$ or, 2

If $C_1 = 3$; $C_2 = 5 - 3 = 2$ and if $C_1 = 2$; $C_2 = 5 - 2 = 3$

So, the capacitances of the two capacitors are $3\,\mu F$ and $2\,\mu F$.

5. Three capacitors having capacitances $1\,\mu F$, $2\,\mu F$ and $3\,\mu F$ are joined in series. A potential difference of 1100 V is applied to the combination. Find the charge and potential difference across each capacitor. (2005)

Solution : If C be the equivalent capacitance of the combination, then,

$$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6} \quad \text{or, } C = \frac{6}{11}\mu F = \frac{6}{11} \times 10^{-6} F$$

$\therefore$ Total charge of the combination,

$$Q = CV = \frac{6}{11} \times 10^{-6} \times 1100 = 6 \times 10^{-4} C$$

Since the capacitors are connected in series, charge on each capacitor is equal to the total charge, i.e., $6 \times 10^{-4}C$.

Potential difference across the plates of the first capacitor,

$$V_1 = \frac{Q}{C_1} = \frac{6 \times 10^{-4}}{1 \times 10^{-6}} = 600V$$

Similarly, for the other two capacitors, respectively,

$$V_2 = \frac{Q}{C_2} = \frac{6 \times 10^{-4}}{2 \times 10^{-6}} = 300V$$

and $V_3 = \dfrac{Q}{C_3} = \dfrac{6 \times 10^{-4}}{3 \times 10^{-6}} = 200V$

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