ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-15

SUBJECT – MATHEMATICS

Pre-test

Chapter: Differentiation **Class: XII**

Topic: Differentiation **Date: 20.06.2020**

SOLVED EXAMPLES

1. $\frac{d}{dx}\left(3^x \log x\right) =$

Solution:

Consider $y = 3^{x\log x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(3^{x\log x}\right)$$

$$= 3^{x\log x} \times \log 3 \frac{d}{dx}(x\log x)$$

[using chain rule]

$$= 3^{x\log x} \times \log 3\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right]$$

[using chain rule]

$$= 3^{x\log x} \times \log 3\left[\frac{x}{x} + \log x\right]$$

$$= 3^{x\log x}(1 + \log x) \times \log 3$$

Hence the sollution is, $\boxed{\frac{d}{dx}\left(3^x \log x\right) = \log 3 \times 3^{x\log x}(1 + \log x)}$

2 $\frac{d}{dx}(\log \sin x)^2 =$

Solution:

Consider $y = (\log \sin x)^2$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(\log \sin x)^2$$

$$= 2(\log \sin x) \times \frac{d}{dx}(\log \sin x)$$

[using chain rule]

$$= 2(\log \sin x) \times \frac{1}{\sin x}\frac{d}{dx}(\log x)$$

$$= 2(\log \sin x) \times \frac{1}{\sin x} \times \frac{1}{x}$$

$$= \frac{2 \log \sin x}{x \sin x}$$

Hence the sollution is, $\boxed{\frac{d}{dx}(\log \sin x)^2 = \frac{2 \log \sin x}{x \sin x}}$

3. $\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) =$

Solution:

Consider $y = e^{\sqrt{\cot x}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{(\cot x)^{\frac{1}{2}}}\right)$$

$$= e^{(\cot x)^{\frac{1}{2}}} \times \frac{d}{dx}(\cot x)^{\frac{1}{2}}$$

[using chain rule]

$$= e^{\sqrt{\cot x}} \times \frac{1}{2}(\cot x)^{\frac{1}{2}-1} \frac{d}{dx}(\cot x)$$

$$= -\frac{e^{\sqrt{\cot x}} \times \cos ec^2 x}{2\sqrt{\cot x}}$$

Hence the sollution is $\boxed{\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = -\frac{e^{\sqrt{\cot x}} \times \cos ec^2 x}{2\sqrt{\cot x}}}$

4. $\frac{d}{dx}\left(\tan^{-1} e^{x}\right) =$

Solution:

Consider $y = \tan^{-1}\left(e^{x}\right)$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} e^{x}\right)$$

$$= \frac{1}{1+\left(e^{2x}\right)^{2}} \frac{d}{dx}\left(e^{x}\right)$$

[using chain rule]

$$= \frac{1}{1+e^{2x}} \times e^{x}$$

$$= \frac{e^{x}}{1+e^{2x}}$$

Hence the sollution is, $\boxed{\frac{d}{dx}\left(\tan^{-1} e^{x}\right) = \frac{e^{x}}{1+e^{2x}}}$

5. $\frac{d}{dx}\left(e^{\sin -1} 2x\right) =$

Solution:

Consider $y = e^{\sin^{-1} 2x}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} 2x}\right)$$

$$= e^{\sin^{-1} 2x} \times \frac{d}{dx}\left(\sin^{-1} 2x\right)$$

[using chain rule]

$$= e^{\sin^{-1} 2x} \times \frac{1}{\sqrt{1-(2x)^{2}}} \frac{d}{dx}(2x)$$

$$= \frac{2e^{\sin -1} 2x}{\sqrt{1-4x^{2}}}$$

Hence the sollution is, $\boxed{\frac{d}{dx}\left(e^{\sin -1} 2x\right) = \frac{2e \sin^{-1} 2x}{\sqrt{1-4x^{2}}}}$

6. $\frac{d}{dx}\left(e^{x}\log\sin 2x\right)=$

Solution:

Consider $y = e^{x}\ \log\sin 2x$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{x}\log\sin 2x\right]$$

$$= e^{2}\frac{d}{dx}\log\sin 2x + \log\sin 2x\frac{d}{dx}\left(e^{x}\right)$$

[using product rule and chain rule]

$$= e^{x}\frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \log\sin 2x\left(e^{x}\right)$$

$$= \frac{e^{x}}{\sin 2x}\cos 2x\frac{d}{dx}(2x) + e^{z}\ \log\sin 2x$$

$$= \frac{2\cos 2xe^{x}}{\sin 2x} + e^{x}\ \log\sin 2x$$

$$= e^{x}\left(2\cot 2x + \log\sin 2x\right)$$

Hence the sollution is, $\boxed{\frac{d}{dx}\left(e^{x}\log\sin 2x\right) = e^{x}\left(2\cot 2x + \log\sin 2x\right)}$

7. Find $\frac{dy}{dx}$:

$xy + y^{2} = \tan x + y$

Answer :

The given relationship is $xy + y^{2} = \tan x + y$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}\left(xy + y^{2}\right) = \frac{d}{dx}(\tan x + y)$$

$$\Rightarrow \frac{d}{dx}(xy) + \frac{d}{dx}\left(y^{2}\right) = \frac{d}{dx}(\tan x) + \frac{dy}{dx}$$

$$\Rightarrow \left[y\cdot\frac{d}{dx}(x) + x.\frac{dy}{dx}\right] + 2y\frac{dy}{dx} = \sec^{2}x + \frac{dy}{dx} \qquad \text{[Using product rule and chain rule]}$$

$$\Rightarrow y\cdot 1 + x.\frac{dy}{dx} + 2y\frac{dy}{dx} = \sec^{2}x + \frac{dy}{dx}$$

$$\Rightarrow (x + 2y - 1)\frac{dy}{dx} = \sec^{2}x - y$$

$$\therefore \frac{dy}{dx} = \frac{\sec^{2}x - y}{(x + 2y - 1)}$$

8.

Find $\frac{dy}{dx}$:

$$y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Answer :

The given relationship is $y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$y=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$
$$\Rightarrow \sin y=\frac{2x}{1+x^2}$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(\sin y)=\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$
$$\Rightarrow \cos y\frac{dy}{dx}=\frac{d}{dx}\left(\frac{2x}{1+x^2}\right) \quad ...(1)$$

The function, $\frac{2x}{1+x^2}$, is of the form of $\frac{u}{v}$.

Therefore, by quotient rule, we obtain

$$\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)=\frac{(1+x^2).\frac{d}{dx}(2x)-2x\cdot\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$
$$=\frac{(1+x^2).2-2x\cdot[0+2x]}{(1+x^2)^2}=\frac{2+2x^2-4x^2}{(1+x^2)^2}=\frac{2(1-x^2)}{(1+x^2)^2} \quad ...(2)$$

Also, $\sin y=\frac{2x}{1+x^2}$

$$\Rightarrow \cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}=\sqrt{\frac{(1+x^2)^2-4x^2}{(1+x^2)^2}}$$
$$=\sqrt{\frac{(1-x^2)^2}{(1+x^2)^2}}=\frac{1-x^2}{1+x^2} \quad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{1-x^2}{1+x^2}\times\frac{dy}{dx}=\frac{2(1-x^2)}{(1+x^2)^2}$$
$$\Rightarrow \frac{dy}{dx}=\frac{2}{1+x^2}$$

9.

Find $\frac{dy}{dx}$:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Answer :

The given relationship is $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$$\Rightarrow \tan y = \frac{3x - x^3}{1 - 3x^2} \qquad \ldots(1)$$

It is known that, $\tan y = \dfrac{3\tan\frac{y}{3} - \tan^3\frac{y}{3}}{1 - 3\tan^2\frac{y}{3}} \qquad \ldots(2)$

Comparing equations (1) and (2), we obtain

$$x = \tan\frac{y}{3}$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(x) = \frac{d}{dx}\left(\tan\frac{y}{3}\right)$$

$$\Rightarrow 1 = \sec^2\frac{y}{3}\cdot\frac{d}{dx}\left(\frac{y}{3}\right)$$

$$\Rightarrow 1 = \sec^2\frac{y}{3}\cdot\frac{1}{3}\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{\sec^2\frac{y}{3}} = \frac{3}{1 + \tan^2\frac{y}{3}}$$

$$\therefore \frac{dy}{dx} = \frac{3}{1 + x^2}$$

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