# ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

**STUDY MATERIAL-12**

**SUBJECT – STATISTICS**

**Pre-test**

**Chapter: THEORITICAL PROBABILITY DISTRIBUTION** **Class: XII**

**Topic: BINOMIAL PROBABILITY DISTRIBUTION** **Date: 24.06.20**

# PROBABILITY DISTRIBUTION

# PART 6

1. If a person tosses an unbiased coin 2n times, then find the probability of getting number of heads as multiple of two.

Solution:

X : Number of heads appear in 2n tosses of an unbiased coin.

So X ~ Bin ( 2n, $\frac{1}{2}$ )

$(1+x)^{2n} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots . + c_{2n} x^{2n}$ ……(1)

Where $c_r = 2n_{c_r}$ $. r = 1, 2, \ldots, 2n$

Putting x = $-1$, in (1)

$(1-1)^{2n} = c_0 - c_1 + c_2 - c_3 + \ldots . + c_{2n}$ ……………(2)

Putting x = $+1$, in (1)

$(1+1)^{2n} = c_0 + c_1 + c_2 + c_3 + \ldots . + c_{2n}$ ……………(3)

Now, adding (2) and (3)

$(2)^{2n} = 2\{ c_0 + c_2 + c_4 + \ldots . + c_{2n}\}$

Hence, P( X = multiple of 2) = $\{ c_0 + c_2 + c_4 + \ldots . + c_{2n}\}(\frac{1}{2})^{2n}$

$$= \frac{1}{2}\, 2^{2n} (\frac{1}{2})^{2n} = \frac{1}{2}.$$

2. If a person tosses an unbiased coin 3n times, then find the probability of getting number of heads as multiple of three.

Solution:

X : Number of heads appear in 3n tosses of an unbiased coin.

So X ~ Bin ( 3n, $\frac{1}{2}$ )

$(1+x)^{3n} = c_0 + c_1x + c_2x^2 + c_3x^3 + \ldots. + c_{3n}x^{3n}$ ……(1)

Where $c_r = 3n_{c_r}$ $.r = 1, 2, \ldots, 3n$

Putting x = $+1$, in (1)

$(1+1)^{3n} = c$

$(2)^{3n} = c_0 + c_1 + c_2 + c_3 + \ldots. + c_{3n}$ …………(2)

Putting x = $w$, in (1), where $w$ = cube root of unity

$(1+w)^{3n} = c_0 + c_1w + c_2w^2 + c_3w^3 + \ldots. + c_{3n}w^{3n}$

$\Rightarrow (1+w)^{3n} = c_0 + c_1w + c_2w^2 + c_3 + \ldots. + c_{3n}$ ………(3)

Putting x = $w^2$, in (1), where $w$ = cube root of unity

$(1+w^2)^{3n} = c_0 + c_1w^2 + c_2w^4 + c_3w^6 + \ldots. + c_{3n}w^{6n}$

$\Rightarrow (1+w^2)^{3n} = c_0 + c_1w^2 + c_2w^2 + c_3 + \ldots. + c_{3n}$ ………(4)

Now adding (2), (3) and (4) we get

$(2)^{3n} + (1+w)^{3n} + (1+w^2)^{3n}$ = 3{ $c_0 + c_3 + c_6 + \ldots. + c_{3n}$}

……….(5)

Now put $w = \frac{-1+i\sqrt{3}}{2} \Rightarrow w^2 = \frac{-1-i\sqrt{3}}{2}$ in (5)

Now L.H.S. in (5) = $(2)^{3n} + (1 + \frac{-1+i\sqrt{3}}{2})^{3n} + (1 + \frac{-1+i\sqrt{3}}{2})^{3n}$

$$= (2)^{3n} + (\frac{-1}{2} + i\frac{\sqrt{3}}{2})^{3n} + (\frac{-1}{2} - i\frac{\sqrt{3}}{2})^{3n}$$

$$= (2)^{3n} + (cos\frac{\pi}{3} + i\, sin\frac{\pi}{3})^{3n} + (cos\frac{\pi}{3} - i\, sin\frac{\pi}{3})^{3n}$$

$$= (2)^{3n} + \left(cos\frac{3n\pi}{3} + i\, sin\frac{3n\pi}{3}\right) + \left(cos\frac{3n\pi}{3} + i\, sin\frac{3n\pi}{3}\right)$$

( By De Moivre's theorem)

$$= (2)^{3n} + 2\cos n\pi$$

So the required probability = $\frac{(2^{3n}-2)}{3}\ \frac{1}{2^{3n}}$ when n is odd

$= \frac{(2^{3n}+2)}{3}\ \frac{1}{2^{3n}}$ when n is even.

3. If a person tosses an unbiased coin 4n times, then find the probability of getting number of heads as multiple of four.

Solution:

X : Number of heads appear in 4n tosses of an unbiased coin.

So X ~ Bin ( 4n, $\frac{1}{2}$ )

$$(1+x)^{4n} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots. + c_{4n} x^{4n} \qquad \ldots\ldots(1)$$

Where $c_r = 4n_{c_r}$ $. r = 1, 2, \ldots, 4n$

Putting x = $+1$, in (1)

$$(1+1)^{3n} = c$$

$$(2)^{4n} = c_0 + c_1 + c_2 + c_3 + \ldots. + c_{4n} \qquad \ldots\ldots\ldots\ldots(2)$$

Putting x = $-1$, in (1),

$$(1-1)^{4n} = c_0 - c_1 + c_2 - c_3 + \ldots. + c_{4n}$$

$$\Rightarrow 0 = c_0 - c_1 + c_2 - c_3 + \ldots. + c_{4n} \qquad \ldots\ldots\ldots(3)$$

Putting x = $i$, in (1),

$$(1+i)^{4n} = c_0 + c_1 i + c_2 i^2 + c_3 i^3 + \ldots. + c_{4n} i^{4n}$$

$$\Rightarrow (1+i)^{4n} = c_0 + c_1 i - c_2 - c_3 i + \ldots. + c_{4n} \qquad \ldots\ldots\ldots(4)$$

Putting x = $-i$, in (1),

$$(1-i)^{4n} = c_0 - c_1 i + c_2 i^2 - c_3 i^3 + \ldots. + c_{4n} i^{4n}$$

$$\Rightarrow (1-i)^{4n} = c_0 - c_1 i + c_2 + c_3 i + \ldots. + c_{4n} \qquad \ldots\ldots\ldots(5)$$

Now adding (2), (3),(4) and (5) we get

$$(2)^{4n} + (1+i)^{4n} + (1-i)^{4n} = 4\{ c_0 + c_4 + c_4 + \ldots. + c_{4n}\}$$

$$\ldots\ldots\ldots\ldots(6)$$

Now the required probability

$$= \frac{1}{4}\left((2)^{4n} + (1+i)^{4n} + (1-i)^{4n}\right)\frac{1}{2^{4n}}$$

$$= \frac{1}{2^{2n+2}}\left(\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{4n} + \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{4n}\right) + \frac{1}{4}$$

$$= \frac{1}{4} + \frac{1}{2^{2n+2}}\left(\left(cos\frac{\pi}{4} + i\ sin\frac{\pi}{4}\right)^{4n} + \left(cos\frac{\pi}{4} - i\ sin\frac{\pi}{4}\right)^{4n}\right)$$

( By De Moivre's theorem)

$$= \frac{1}{4} + \frac{1}{2^{2n+2}}\ 2\cos n\pi$$

$$= \frac{1}{4} + \frac{1}{2^{2n+1}}\ when\ n\ is\ even$$

$$= \frac{1}{4} - \frac{1}{2^{2n+1}}\ when\ n\ is\ odd$$

**Note:** In case of a Bin(n, p) to find the probability that number of success is multiple of r(any positive integer), then we need to put x as rth roots of unity in the expansion of $(1+x)^n$ and add them.

4. A random variable X~ $bin(n, p)$

i. Find the probability distribution that x throws are required to get the first success.

ii. Find the probability distribution that x throws are required to get the m successes.

Solution:

X~ $bin(n,p)$

So the pmf is given by

$$f(x) = n_{c_x} p^x (1-p)^{n-x} , x = 0(1)n$$

i. Required probability distribution

$$f(x) = n_{c_{x-1}} p^{x-1} (1-p)^{n-x+1} p , x = 1(1)n$$

$$= n_{c_{x-1}} p^x (1-p)^{n-x+1} , x = 1(1)n$$

ii. . Required probability distribution

$$f(x) = x-1_{c_{m-1}} p^{m-1} (1-p)^{x-m} p , x = m(1)n$$

$$= x-1_{c_{m-1}} p^m (1-p)^{x-m} , x = m(1)n$$

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