ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

Sub: Arithmetic **Class: 7** **Date: 04.05.20**

STUDY MATERIAL: EXPONENTS

Important Formulae

Laws of Exponents

Powers with like bases

- $a^n \times a^m = a^{n+m}$.
  Example: $3^2 \times 3^4 = 3^6 = 729$
- $\frac{a^n}{a^m} = a^{n-m}$.
  Example: $2^5 \div 2^3 = \frac{32}{8} = 4 = 2^2$
- $a^m \times a^{-m} = a^m \times \frac{1}{a^m} = 1$

Power of a Power

- $(a^n)^m = a^{nm}$

Exponent Zero

- $a^m \times \frac{1}{a^m} = 1$
  $\Rightarrow \frac{a^m}{a^m} = a^{m-m} = a^0 = 1$

Powers with unlike bases and same exponent

- $a^n \times b^n = (ab)^n$

$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$

Concepts

Introduction

Powers and Exponents

- Repeated multiplication of the same number can be expressed in the form of exponents.
- Example: $625 = 5 \times 5 \times 5 \times 5 \ or \ 5^4$.
  Here '5' is the base raised to the power of 4, where 4 is the exponent and 5^4 is the exponential form of 625.

Powers with negative exponents

- Numbers can have positive powers which are called positive index. Example $a^n = a \times a \times a....$ n times.
- Numbers can also have negative powers such as $a^{-m} = \frac{1}{a^m} = \frac{1}{a \times a \times a.............m\ times}$
- Example : $5^{-3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125} = 0.008$

Visualising Exponents

Visualising powers and exponents

- Example 1: 54 can be expressed as product of powers of prime numbers.
  $$54 = 2 \times 3 \times 3 \times 3 = 3^3 \times 2^1$$
- Example 2 :We know that $6^4 < 4^6$. This can be visualised as shown below:
  $$6^4 = 6 \times 6 \times 6 \times 6 = 1296$$
  $$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$$
  $$\therefore 6^4 < 4^6$$

Uses of Exponents

Expanding a rational number using powers

- Rational Numbers can be expanded using exponents and powers.
- Example 1: 1284 can be written as $1 \times 10^3 + 2 \times 10^2 + 8 \times 10^1 + 4 \times 10^0$.
- Example 2: 0.597 can be written as $5 \times 10^{-1} + 9 \times 10^{-2} + 7 \times 10^{-3}$.

Inter conversion between standard and normal forms

- Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its **standard form**.
- Example:
  $43 = 4.3 \times 10 = 4.3 \times 10^1$
  $430 = 4.3 \times 100 = 4.3 \times 10^2$
  $4300 = 4.3 \times 1000 = 4.3 \times 10^3$
  $43000 = 4.3 \times 10000 = 4.3 \times 10^4$

Comparison of quantities using exponents

- If two numbers in standard form have the same power of 10, then the number with the larger factor is greater.
  E.g : $2.05 \times 10^3 > 1.05 \times 10^3$
- If two numbers in standard form have the same factor, then the number with the larger power of 10 will be greater.
  E.g $2.05 \times 10^6 > 2.05 \times 10^3$

Solved Numericals

1. Write the following using exponents:

   a) 8 × 8 × 8 × 8

   b) 10 × 10 × 10 × 10 × 10 × 10 × 10

   c) A × A × A

d) meter × meter

e) centimeter × centimeter × centimeter

Solution

Use the definition of an exponent.

a) $8 \times 8 \times 8 \times 8 = 8^4$, 8 multiplied by itself 4 times.

b) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^7$

c) $A \times A \times A = A^3$

d) meter × meter = meter ^{2}

e) centimeter × centimeter × centimeter = centimeter ^{3}

2. Evaluate the following:

a) 2^4

b) 10^4

c) $(-2)^4$

d) -2^4

Solution

Use the definition of an exponent.

a) $2^4 = 2 \times 2 \times 2 \times 2 = 16$

b) $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

c) $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

d) $-2^4 = -2 \times 2 \times 2 \times 2 = -16$

3. Use exponents to write the following using one power only.

a) 4×8

b) 25×5

c) $16 \times 4 \times 4^3$

d) $2 \times 2 \times 8 \times 2^3$

e) $B \times B \times B^3$

<u>Solution</u>

a) Express 4 and 8 as products of 2 then use exponents.

$4 \times 8 = (2 \times 2) \times (2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

b) Express 25 as products of 5 then use exponents.

$25 \times 5 = (5 \times 5) \times 5 = 5 \times 5 \times 5 = 5^3$

c) Express all terms as products of 4 then use exponents.

$16 \times 4 \times 4^3 = (4 \times 4) \times 4 \times (4 \times 4 \times 4) = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

d) Express all terms as products of 2 then use exponents.

$2 \times 2 \times 8 \times 2^3 = 2 \times 2 \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

e) Express all terms as products of B then use exponents.

$B \times B \times B^3 = B \times B \times (B \times B \times B) = B \times B \times B \times B \times B = B^5$

4. Use exponents to rewrite the following expressions in simplified forms.

a) $2^3 \times 2^4$

b) 6×6^3

c) $5 \times 5^2 \times 5^3$

Solution

a) Use the definition of exponents to write expressions as product of 2 then rewrite as in exponent form.

$2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^7$

b) Use the definition of exponents to write expressions as product of 6 then rewrite as in exponent form.

$6 \times 6^3 = 6 \times (6 \times 6 \times 6) = 6^4$

c) Use the definition of exponents to write expressions as product of 5 then rewrite as in exponent form.

$5 \times 5^2 \times 5^3 = 5 \times (5 \times 5) \times (5\times 5 \times 5) = 5^6$

Solution of Previous Years' Question Papers

2019

1st term

4) Simplify:- $3^8 \div 3^{-2}$

$\frac{3^8}{3^{-2}} = 3^{8-(-2)} = 3^{8+2} = 3^{10} = 59049$

2nd Term

2) Simplify: $(12^2 - 5^3) \times \frac{(-1)^{40}}{19}$

Ans : $(144 - 125) / 19 = 19/19 = 1$

3rd Term

iv) $(-1)^{235} \times (-4)^{11} \times (-10)^9$ results in a :

a) Negative number

4. v) What is the single exponent of $(3^3)^7 \times (3^6)^2$? Ans-3^{33}

1. Simplify: $(2^{-1} \div 5^{-1})^2 \times (\frac{-5}{8})^{-2}$

Ans-16

2018

1st Term

v)The value of $7^0 + 8^0 + 9^0$ is
c) 3

ii)Absolute value of $-(\frac{7}{8})^2$ is $\frac{49}{64}$. TRUE
iii)simplifying (-5) x $(-5)^2$ we get 125.FALSE

(v) $-\frac{1}{32} = -\frac{1}{2^5}$

(viii) $(-\frac{4}{5})^{2+4} = (-\frac{4}{5})^6$ [Since, bases are equal]

3rd Term

(i) The reciprocal of $(-3)^4$ is $(\frac{-1}{3})^4$

ii	$(\frac{p}{q})^n$	a	$\frac{p^n}{q^n}$

(iv) Find the reciprocal of $[(\frac{1}{3})^{-3} - (\frac{1}{2})^{-3}] \div (\frac{1}{4})^{-3}$
$(27 - 8) \div (4)^3 = \frac{19}{64}$; ∴ Reciprocal is $\frac{64}{19}$

Exercise Problems

Question 1.
Fill in the blanks:-
i. Any non-zero number with exponent zero is equal to ______________
ii. $2^x=16$, then x is ________
iii. Writing repeated multiplication of numbers in a short form is called ____________________
iv. a raised to the power 2 is called a ________________ and a raised to the power 3 is called a ____________
v. if base is negative and exponent is odd, the expression has _______________ value.

Question 2.
State true or false:-
i. $(a\div a)\times a=a$; for every non-zero rational number
ii. $a^5\times b^5=ab^5$
iii. $b^{12}\times(-b)^6=b^{18}$
iv. 2 is the base of exponential expression $(-2)^{10}$.
v. $2^4\times2^3=6^7$
vi . $1^0\times0^1=1$
vii. $3^4>4^3$
viii. $4^0+5^0+6^0=(4+5+6)^0$
Question 3
if $4^x=64$, then find the value of 12^{2x-6}

Question 4.
Find x if
i. $11^6 \div 11^{4-x} = 11^8$
ii. $(-3)^{x-2} = -243$

Question 5.
Simplify and express as a power of a rational number.
i. $(4^3)^2 \times (2^3)^3 \times 14$
ii. $[(-3^5)^2 \times (-3^5)^4]^3$
iii. $3^5 \times 10^5 \times 255^7 \times 6^5$
iv. $3^8 \times a^6 9^3 \times a^3$

Question 6.
Evaluate:-
i. $(9^0 \times 7^0) \times (9+7)$
ii. $(-8)^0 - 11^0 (-23)^0$
iii.$(-1)^{10} \times (-1)^5 \times (-1)^4$
iv. $(-1)^{12} - (-1)^7$

Question 7.
Write in standard form;
i) 7489.3 ii) 6,780,000
iii) 8,04,000 iv) 78,950

Question 8
a is a non-zero rational number. Product of the square of a with the cube of a is equal to the
(a) second power of a
(b) third power of a
(c) fifth power of a
(d) sixth power of a

Question 9
Which of the follwing is equal to 1?
(a) $2^0 + 3^0 + 4^0$
(b) $2^0 \times 3^0 \times 4^0$
(c) $(3^0 - 2^0) \times 4^0$
(d) $(3^0 - 2^0) \times (3^0 + 2^0)$

Question 10
Which of the two is larger : 3^{12} or 6^6 ?