ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-4

SUBJECT – STATISTICS

1st term

Chapter: INTERPOLATION **Class: XI**

Topic: Newtons backward and Lagrange's formula **Date: 26.06.20**

INTERPOLATION

PART 2

Consider x: arguments and y: entry and y=f(x).

Given n+1 pairs of values of arguments and corresponding entries the polynomial can be formed, is of degree n.

Assumption: all the arguments are equidistant, i.e, those are in A.P.

Let us consider h= the common difference or the common interval between the successive arguments.

The argument for which the the entry to be found should be in the lower half of the given values.

NEWTON'S BACKWARD INTERPOLATION FORM

Given, Argument(x): $x_0 \;\; x_1 \;\; x_2 \;\; .... \;\; x_n$

Entry (y): $y_0 \;\; y_1 \;\; y_2 \;\; .... \;\; y_n$

h : the common difference of arguments and $y = f(x)$

Let the polynomial be

$$f(x) = b_0 + b_1(x - x_n) + b_2(x - x_n)(x - x_{n-1}) + b_3(x - x_n)(x - x_{n-1})(x - x_{n-2})$$
$$+ \cdots + b_n(x - x_n)(x - x_{n-1}) \ldots (x - x_1)$$

Put $x = x_n, f(x_n) = b_0 \;\Rightarrow\; y_n = b_0$

Put $x = x_{n-1}, f(x_{n-1}) = b_n + \; b_{n-1}(x_{n-1} - x_n)$

$\Rightarrow\; y_{n-1} = y_n - b_1.h$

$\Rightarrow\; b_1 = \frac{y_n - y_{n-1}}{h} \;=\frac{\Delta y_{n-1}}{h}$

Put $x = x_{n-2}$, $f(x_{n-2}) = b_0 + b_1(x_{n-2} - x_n) + b_2(x_{n-2} - x_n)(x_{n-2} - x_{n-1})$

$\Rightarrow\ y_{n-2} = y_n - \frac{y_n - y_{n-1}}{h}.2h + a_2(-2h).(-h)$

$\Rightarrow\ y_{n-2} = y_n - 2\,(y_n - y_{n-1}) + a_2(2!)h.h$

$\Rightarrow\ b_2 = \frac{y_n - 2y_{n-1} + y_{n-2}}{2!\,h^2} = \frac{\Delta^2 y_{n-2}}{2!h^2}$

Similarly,

$$b_r = \frac{\Delta^r y_{n-r}}{r!h^r} \quad \text{r=1(1) n}$$

Substituting the values of b_r in the equation

$$f(x) = y_n + \frac{\Delta y_{n-1}}{h}(x - x_n) + \frac{\Delta^2 y_{n-2}}{2!\,h^2}(x - x_n)(x - x_{n-1}) + \frac{\Delta^3 y_{n-3}}{3!\,h^3}(x - x_n)(x - x_{n-1})(x - x_{n-2}) + \cdots + \frac{\Delta^n y_0}{n!h^n}(x - x_n)(x - x_{n-1}) \ldots (x - x_1) \quad \ldots\ldots\ldots\ldots(2)$$

Take $v = \frac{x - x_n}{h}$

Then $v + k = \frac{x - x_n}{h} + k = \frac{x - x_n + kh}{h} = \frac{x - x_n + (x_n - x_{n-k})}{h} = \frac{x - x_{n-k}}{h}$

So from (2),

$$f(x) = y_n + v\,\Delta y_{n-1} + \frac{v(v+1)}{2!}\Delta^2 y_{n-2} + \frac{v(v+1)(v+2)}{3!}\Delta^3 y_{n-3} + \ldots + \frac{v(v+1)\ldots(v+n-1)}{n!}\Delta^n y_0$$

Which is known as Newton' backward interpolation formula.

LAGRANGES INTERPOLATION FORMULA

This formula is being used if the arguments are not equidistant.

Given, Argument(x): $x_0 \;\; x_1 \;\; x_2 \;\; \ldots. \;\; x_n$

Entry (y): $y_0 \;\; y_1 \;\; y_2 \;\; \ldots. \;\; y_n$

Here we assume the nth degree polynomial as

$$f(x) = c_0(x - x_1)(x - x_2)(x - x_3) \ldots \ldots \ldots .(x - x_n)$$

$$+ c_1(x - x_0)(x - x_2)(x - x_3) \ldots \ldots \ldots .(x - x_n)$$

$$+ c_2(x - x_0)(x - x_1)(x - x_3) \ldots \ldots \ldots .(x - x_n)$$

$$+ \ldots\ldots\ldots\ldots$$

$$+ c_n(x - x_0)(x - x_1)(x - x_2) \ldots \ldots \ldots .(x - x_{n-1})$$

Put $x = x_0$,

$$f(y_0) = c_0(x_0 - x_1)(x_0 - x_2)(x_0 - x_3) \ldots \ldots \ldots .(x_0 - x_n)$$

$$\Rightarrow c_0 = y_0 . \frac{1}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3) \ldots \ldots \ldots .(x_0 - x_n)}$$

Put $x = x_1$,

$f(x_1) = c_1(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\ldots\ldots\ldots.(x_1 - x_n)$

$$\Rightarrow c_1 = y_1.\frac{1}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)\ldots\ldots\ldots.(x_1 - x_n)}$$

Put $x = x_2$,

$f(x_2) = c_2(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)\ldots\ldots\ldots.(x_2 - x_n)$

$$\Rightarrow c_2 = y_2.\frac{1}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)\ldots\ldots\ldots.(x_2 - x_n)}$$

Put $x = x_3$,

$f(x_3) = c_3(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)\ldots\ldots\ldots.(x_3 - x_n)$

$$\Rightarrow c_3 = y_3.\frac{1}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)\ldots\ldots\ldots.(x_3 - x_n)}$$

....

Put $x = x_n$,

$f(x_n) = c_n(x_n - x_0)(x_n - x_1)(x_n - x_2)\ldots\ldots\ldots.(x_n - x_{n-1})$

$$\Rightarrow c_n = y_n.\frac{1}{(x_n - x_0)(x_n - x_1)(x_n - x_2)\ldots\ldots\ldots.(x_n - x_{n-1})}$$

So substituting $c_{0,}c_{1,}\ c_{2,} \dots \dots \dots , c_n$

$$f(x) = y_0 . \frac{(x-x_1)(x-x_2)(x-x_3)\dots\dots\dots.(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\dots\dots\dots.(x_0-x_n)}$$

$$+ y_1 . \frac{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots\dots\dots(x_1-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots\dots\dots(x_1-x_n)}$$

$$+ y_2 . \frac{(x-x_0)(x-x_1)(x-x_3)\dots\dots\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots\dots\dots(x_2-x_n)}$$

$$+ \dots\dots\dots\dots$$

$$+ y_n . \frac{(x-x_0)(x-x_1)(x-x_2)\dots\dots\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\dots\dots\dots(x_n-x_{n-1})}$$

Which is the Lagrange's fomula of interpolation.

Prepared by

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