ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-8

SUBJECT – MATHEMATICS

1st term

Chapter: Trigonometry **Class: XI**

Topic: Trigonometric Identities **Date: 26.06.2020**

Trigonometric Ratios and Identities

(Solved MCQs – Set 3) :-

42. If $\sin x+\sin^2 x=1$, then the value of $\cos^2 x+\cos^4 x+\cot^4 x-\cot^2 x$ is equal to

(A) 1
(B) 0
(C) 2
(D) None of these

43. $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{4\pi}{9}\sin\frac{\pi}{3}$ is equal to

(A) $\frac{3}{4}$
(B) $\frac{3}{8}$
(C) $\frac{3}{16}$
(D) None of these

44. If x, y, z are in AP, then $\frac{\sin x-\sin z}{\cos z-\cos x}$ is equal to

(A) $\tan y$
(B) $\cot y$
(C) $\sin y$
(D) $\cos y$

45. If $|\sin x+\cos x|=|\sin x|+|\cos x|$, then x belongs to the quadrant

(A) I or III
(B) II or IV
(C) I or II
(D) III or IV

46. Find the value of x if $x=\frac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$.

(A) 1
(B) 2
(C) 3
(D) None of these

47. Given $\frac{\pi}{2}<\alpha<\pi$, then the expression $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}}+\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$ is equal to

(A) $\frac{1}{\cos\alpha}$
(B) $-\frac{2}{\cos\alpha}$
(C) $\frac{2}{\cos\alpha}$
(D) None of these

48. If $\cot\alpha\cot\beta = 2$, then $\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$ is equal to

(A) $\dfrac{1}{3}$ **(B)** $-\dfrac{1}{3}$

(C) $\dfrac{1}{2}$ **(D)** $-\dfrac{1}{2}$

49. If $\cos(x-y)$, $\cos x$ and $\cos(x+y)$ are in HP, then $\cos x \cdot \sec\dfrac{y}{2}$ is equal to

(A) $\sqrt{2}$ **(B)** $-\sqrt{2}$

(C) $\pm\sqrt{2}$ **(D)** None of these

50. If $\cot\theta = \dfrac{\sqrt{m}+\sqrt{n}}{\sqrt{m}-\sqrt{n}}$, then $(m+n)\cos 2\theta$ is equal to

(A) $2\sqrt{mn}$ **(B)** $\dfrac{m+n}{m-n}$

(C) $(\sqrt{m}+\sqrt{n})^2$ **(D)** None of these

51. If $\cos^2 x + \cos^4 x = 1$, then the value of $\tan^4 x + \cot^4 x + \tan^2 x - \cot^2 x$ is equal to

(A) 0 **(B)** 2

(C) 1 **(D)** None of these

52. If $\cos 25^\circ + \sin 25^\circ = k$, then $\cos 20^\circ$ is equal to

(A) $\dfrac{k}{\sqrt{2}}$ **(B)** $-\dfrac{k}{\sqrt{2}}$

(C) $\pm\dfrac{k}{\sqrt{2}}$ **(D)** None of these

53. If $\tan\theta = n\tan\phi$, then the maximum value of $\tan^2(\theta-\phi)$ is equal to

(A) $\dfrac{(n-1)^2}{4n}$ **(B)** $\dfrac{(n+1)^2}{4n}$

(C) $\dfrac{(n+1)}{2n}$ **(D)** $\dfrac{(n-1)}{2n}$

54. If $a \le 16 \sin x \cos x + 12 \cos^2 x - 6 \le b$ for all $x \in R$, then

(A) $a = -5, b = 5$
(B) $a = -4, b = 4$
(C) $a = -10, b = 10$
(D) None of these

55. Let $f(\theta) = \dfrac{\cot\theta}{1+\cot\theta}$ and $\alpha + \beta = \dfrac{5\pi}{4}$. Then the value of $f(\alpha) \cdot f(\beta)$ is

(A) 2
(B) $-\dfrac{1}{2}$
(C) $\dfrac{1}{2}$
(D) None of these

56. If $\cos\alpha = \dfrac{2\cos\beta - 1}{2 - \cos\beta}$ $(0 < \alpha < \pi,\ 0 < \beta < \pi)$, then $\tan\dfrac{\alpha}{2}\cot\dfrac{\beta}{2}$ is equal to

(A) 1
(B) $\sqrt{2}$
(C) $\sqrt{3}$
(D) None of these

57. The value(s) of y for which the equation $4\sin x + 3\cos x = y^2 - 6y + 14$ has a real solution, is (are)

(A) 3
(B) 5
(C) -3
(D) None of these

-:SOLUTIONS:-

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42. Given $\sin x + \sin^2 x = 1$. So

$$\sin x = \cos^2 x$$

$$\Rightarrow \sin^2 x = \cos^4 x$$

Now,

$$\cos^2 x + \cos^4 x + \cot^4 x - \cot^2 x$$

$$= \cos^2 x + \sin^2 x + \text{cosec}^2 x - \cot^2 x \ = 2$$

43. We have

$$\sin 20° \sin 40° \sin 80° \sin 60°$$

$$= \frac{\sqrt{3}}{2}[\sin 20° \cdot \sin(60° - 20°) \cdot \sin(60° + 20°)]$$

$$= \frac{\sqrt{3}}{2}[\sin 20° \cdot (\sin^2 60° - \sin^2 20°)]$$

$$= \frac{\sqrt{3}}{2}\left[\frac{3\sin 20° - 4\sin^3 20°}{4}\right]$$

$$= \frac{\sqrt{3}}{8} \cdot \sin 60° = \frac{3}{16}$$

44. Given x, y, z are in AP $\Rightarrow 2y = x + z$. Now

$$\frac{\sin x - \sin z}{\cos z - \cos x} = \frac{2\cos\frac{x+z}{2} \cdot \sin\frac{x-z}{2}}{2\sin\frac{x+z}{2} \cdot \sin\frac{x-z}{2}} = \cot y$$

45. Given $|\sin x + \cos x| = |\sin x| + |\cos x|$
Then $\sin x$ and $\cos x$ both will be positive or negative.
Hence, x belongs to I quadrant or III quadrant.

46. We have

$$x = \frac{1 - 4\sin 10° \cdot \sin 70°}{2\sin 10°} = \frac{1 - 2(\sin 30° - \sin 10°)}{2\sin 10°}$$

$$= \frac{1 - 1 + 2\sin 10°}{2\sin 10°} = 1$$

47. We have

$$\sqrt{\frac{1 - \sin\alpha}{1 + \sin\alpha}} + \sqrt{\frac{1 + \sin\alpha}{1 - \sin\alpha}} = \frac{1 - \sin\alpha + 1 + \sin\alpha}{\sqrt{1 - \sin^2\alpha}}$$

$$= \frac{2}{|\cos\alpha|} = -\frac{2}{\cos\alpha} \quad \left(\because \frac{\pi}{2} < \alpha < \pi\right)$$

48. Given $\cot\alpha \cot\beta = 2$. So

$$\frac{\cos\alpha \cdot \cos\beta}{\sin\alpha \cdot \sin\beta} = \frac{2}{1}$$

$$\Rightarrow \frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{1}{3}$$

49. Given $\cos(x - y)$, $\cos x$ and $\cos(x + y)$ are in HP. So

$$\frac{2}{\cos x} = \frac{1}{\cos(x - y)} + \frac{1}{\cos(x + y)}$$

$$\Rightarrow \frac{2}{\cos x} = \frac{2\cos x \cdot \cos y}{\cos^2 x - \sin^2 y}$$

$$\Rightarrow \cos^2 x - \sin^2 y = \cos^2 x \cdot \cos y$$

$$\Rightarrow \cos^2 x \cdot 2\sin^2\frac{y}{2} = 4\sin^2\frac{y}{2} \cdot \cos^2\frac{y}{2}$$

$$\Rightarrow \cos x \cdot \sec\frac{y}{2} = \pm\sqrt{2}$$

50. Given $\tan\theta = \frac{\sqrt{m} - \sqrt{n}}{\sqrt{m} + \sqrt{n}}$. So

$$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \left(\frac{\sqrt{m} - \sqrt{n}}{\sqrt{m} + \sqrt{n}}\right)^2}{1 + \left(\frac{\sqrt{m} - \sqrt{n}}{\sqrt{m} + \sqrt{n}}\right)^2}$$

$$= \frac{4\sqrt{mn}}{2(m + n)}$$

$$\Rightarrow (m + n)\cos 2\theta = 2\sqrt{mn}$$

51. Given $\cos^2 x + \cos^4 x = 1$

$$\Rightarrow \cos^4 x = \sin^2 x \Rightarrow \cot^4 x = \text{cosec}^2 x$$

Now, $\quad \tan^4 x + \cot^4 x + \tan^2 x - \cot^2 x$

$$= \tan^2 x(1 + \tan^2 x) + \text{cosec}^2 x - \cot^2 x$$

$$= \tan^2 x \cdot \sec^2 x + 1$$

$$= \frac{\sin^2 x}{\cos^4 x} + 1 = 2$$

52. Given $\cos 25° + \sin 25° = k$

$$\Rightarrow \cos(45° - 20°) + \sin(45° - 20°) = k$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos 20° + \frac{1}{\sqrt{2}}\sin 20° + \frac{1}{\sqrt{2}}\cos 20° - \frac{1}{\sqrt{2}}\sin 20° = k$$

$$\Rightarrow \cos 20° = \frac{k}{\sqrt{2}}$$

53. We have

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta \cdot \tan\phi} = \frac{(n - 1)\tan\phi}{1 + n\tan^2\phi}$$

$$\Rightarrow \tan^2(\theta - \phi) = \frac{(n - 1)^2}{(\cot\phi + n\tan\phi)^2} = \frac{(n - 1)^2}{(\cot\phi - n\tan\phi)^2 + 4n}$$

$$\Rightarrow \tan^2(\theta-\phi) \le \frac{(n-1)^2}{4n}$$

Therefore, maximum value of $\tan^2(\theta-\phi)$ is $\frac{(n-1)^2}{4n}$.

54. We have

$$16\sin x \cdot \cos x + 12\cos^2 x - 6 = 8\sin 2x + 6\cos 2x$$

Now,

$$-\sqrt{8^2+6^2} \le 8\sin 2x + 6\cos 2x \le \sqrt{8^2+6^2}$$

$$\Rightarrow -10 \le 8\sin 2x + 6\cos 2x \le 10$$

Hence, $a = -10$, $b = 10$.

55. We have

$$f(\alpha)\cdot f(\beta) = f(\alpha)\cdot f\left(\frac{5\pi}{4}-\alpha\right)$$

$$= \frac{\cot\alpha}{1+\cot\alpha}\cdot\frac{\cot\left(\frac{5\pi}{4}-\alpha\right)}{1+\cot\left(\frac{5\pi}{4}-\alpha\right)}$$

$$= \frac{\cot\alpha}{1+\cot\alpha}\cdot\frac{\frac{\cot\alpha+1}{\cot\alpha-1}}{1+\frac{\cot\alpha+1}{\cot\alpha-1}}$$

$$= \frac{\cot\alpha}{1+\cot\alpha}\cdot\frac{\cot\alpha+1}{2\cot\alpha} = \frac{1}{2}$$

56. Given $\cos\alpha = \frac{2\cos\beta-1}{2-\cos\beta}$. So

$$\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{2\cdot\frac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}-1}{2-\frac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}}$$

$$\Rightarrow \quad \frac{1-\tan^2(\alpha/2)}{1+\tan^2(\alpha/2)} = \frac{1-3\tan^2(\beta/2)}{1+3\tan^2(\beta/2)}$$

$$\Rightarrow \quad \tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2} \qquad \text{[By Componendo and Dividendo]}$$

$$\Rightarrow \quad \tan\frac{\alpha}{2}\cdot\cot\frac{\beta}{2} = \sqrt{3}$$

57. Given equation is

$$4\sin x + 3\cos x = y^2 - 6y + 14$$

Now, $\quad -5 \le 4\sin x + 3\cos x \le 5.$

Hence, $-5 \le y^2 - 6y + 14 \le 5$. Now

$$y^2 - 6y + 14 \le 5$$

$$\Rightarrow y^2 - 6y + 9 \le 0 \Rightarrow (y-3)^2 \le 0$$

Hence, $y = 3$.

Again, if

$$y^2 - 6y + 14 \ge -5 \Rightarrow y^2 - 6y + 19 \ge 0$$

which is always true. Thus for $y = 3$, there exists one value of x for which equation is satisfied.

Bibliography

1. **NCERT MATHEMATICS.**
2. **SHARMA R.D. , ISC MATHEMATICS , D.R. PUBLICATIONS (P) LTD.**
3. **DE SOURENDRANATH , MATHEMATICS , CHHAYA PRAKASHANI PVT. LTD.**
4. **AGGARWAL M.L. , UNDERSTANDING MATHEMATICS , ARYA PUBLICATIONS (P) LTD.**
5. **AGGARWAL R.S. , SENIOR SECONDARY SCHOOL MATHEMATICS , BHARATI BHAWAN PUBLISHERS.**
6. https://en.wikipedia.org/
7. https://www.google.co.in/

Prepared by –

SUKUMAR MANDAL, (Asst. Teacher)