ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-7

SUBJECT – STATISTICS

1st term

Chapter: CENTRAL TENDENCY **Class: XI**

Topic: HARMONIC MEAN **Date: 07.07.2020**

CENTRAL TENDENCY

PART 3

Definition :

The sum of all the observations with degree as the reciprocal of number of observations.

It is denoted by x_h .

Case 1 : Ungrouped or raw data

Observations: $x_1, x_2, x_3, \dots \dots \dots, x_n$

$$\bar{x} = \frac{n}{\sum_{i=1}^{n} x_i}$$

Case 2 : Grouped data

Observations: $\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \dots \dots \dots, \mathrm{x}_n$

Frequencies: $f_1, f_2, f_3, \dots \dots \dots ., f_n$

$$\bar{\mathrm{x}} = \frac{\mathrm{N}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\mathrm{f_i}}{\mathrm{x_i}}} \text{ where } \mathrm{N} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f_i}$$

PROPERTIES:

1. Change of origin or base and scale

If $y_i = b\, x_i$, $\forall\, i = 1(1)n$

Then $\bar{y} = b x_h$

Proof: By definition,

For ungrouped data

$$x_h = \frac{n}{\sum_{i=1}^{n} \frac{1}{y_i}}$$

$$= \frac{n}{\sum_{i=1}^{n} (\frac{1}{b\,x_i})}$$

$$= \frac{bn}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

$$= b\,x_h$$

For grouped data

$$\bar{y} = \frac{N}{\sum_{i=1}^{n} \frac{f_i}{y_i}}$$

$$= \frac{N}{\sum_{i=1}^{n} \frac{f_i}{b\,x_i}}$$

$$= \frac{N}{\frac{1}{b}\sum_{i=1}^{n}\frac{f_i}{x_i}}$$

$$= bx_h$$

2. If all the observations are equal to a constant then the hm is equal to the same constant.

If $x_i = k, \forall\, i = 1(1)n$

Proof:

For ungrouped data

$$x_h = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

$$= \frac{n}{\sum_{i=1}^{n}\frac{1}{k}}$$

$$= \frac{n}{n\frac{1}{k}}$$

$$= k$$

For grouped data

$$x_h = \frac{N}{\sum_{i=1}^{n} \frac{f_i}{x_i}}$$

$$= \frac{N}{\sum_{i=1}^{n} \frac{f_i}{k}}$$

$$= \frac{N}{\frac{1}{N}k} = \text{k}$$

3. HM.of all the observations lies between the minimum and maximum observations.

Proof:

Observation: $x_1, x_2, x_3, \dots \dots \dots, x_n$

To show $x_{(1)} \leq x_h \leq x_{(n)}$

$x_{(1)} = \min\{ x_1, x_2, x_3, \dots \dots \dots, x_n\}$ and

$x_{(n)} = max\ x_1, x_2, x_3, \dots \dots \dots, x_n\}$

For ungrouped data

$$x_h = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} \leq \frac{n}{\sum_{i=1}^{n} \frac{1}{x_{(n)}}} = x_{(n)}$$

$$x_h = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} \geq \frac{n}{\sum_{i=1}^{n} \frac{1}{x_{(1)}}} = x_{(1)}$$

For grouped data

$$x_h = \frac{N}{\sum_{i=1}^{n} \frac{f_i}{x_i}} \leq \frac{N}{\sum_{i=1}^{n} \frac{f_i}{x_{(n)}}} = \frac{N}{N \frac{1}{x_{(n)}}} = x_{(n)}$$

$$x_h = \frac{N}{\sum_{i=1}^{n} \frac{f_i}{x_i}} \geq \frac{N}{\sum_{i=1}^{n} \frac{f_i}{x_{(n)}}} = \frac{N}{N \frac{1}{x_{(n)}}} = x_{(n)}$$

Combining we can say,

$$x_{(1)} \leq x_h \leq x_{(n)}$$

4. Combined or composite harmonic mean

Set 1: observation: $x_{11}, x_{12}, x_{13}, \dots \dots \dots, x_{1n_1}$ with hm= x_{1h}

Set 2: observation: $x_{21}, x_{22}, x_{23}, \dots \dots \dots, x_{2n_2}$ with hm= x_{2h}

Then the combined HM, $\bar{X} = \frac{n_1 + n_2}{n_1 \frac{1}{x_{1h}} + n_2 \frac{1}{x_{2h}}}$

Proof:

By definition,

$$X_h = \frac{n_1 + n_2}{\sum_{i=1}^{n1} \frac{1}{x_{1i}} + \sum_{i=1}^{n2} \frac{1}{x_{2i}}}$$

$$X_h = \frac{n_1 + n_2}{n_1 \frac{1}{x_{1h}} + n_2 \frac{1}{x_{2h}}}$$

Since, $x_{1h} = \frac{n_1}{\sum_{i=1}^{n1} \frac{1}{x_{1i}}}$ and $x_{2h} = \frac{n_2}{\sum_{i=1}^{n2} \frac{1}{x_{2i}}}$

5. Then the combined HM, X_h lies between x_{1h} and x_{2h}

Proof:

Without loss of generality take $x_{1h} < x_{2h}$

$$X_h = \frac{n_1 + n_2}{n_1 \frac{1}{x_{1h}} + n_2 \frac{1}{x_{2h}}} < \frac{n_1 + n_2}{n_1 \frac{1}{x_{2h}} + n_2 \frac{1}{x_{2h}}} = x_{2h}$$

$$X_h = \frac{n_1 + n_2}{n_1 \frac{1}{x_{1h}} + n_2 \frac{1}{x_{2h}}} > \frac{n_1 + n_2}{n_1 \frac{1}{x_{1h}} + n_2 \frac{1}{x_{1h}}} = x_{1h}$$

Hence $x_{1h} < X_h < x_{2h}$

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