# ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

**STUDY MATERIAL-7**

**SUBJECT – MATHEMATICS**

**1st term**

**Chapter: Trigonometry** **Class: XI**

**Topic: Trigonometric Identities** **Date: 25.06.2020**

# Trigonometric Ratios and Identities

# (Solved MCQs – Set 2) :-

**24.** If $\cos^2\theta = \frac{1}{3}(a^2-1)$ and $\tan^2\frac{\theta}{2} = \tan^{2/3}\alpha$, then $\sin^{2/3}\alpha + \cos^{2/3}\alpha =$

(A) $2a^{2/3}$

(B) $\left(\frac{2}{a}\right)^{2/3}$

(C) $\left(\frac{2}{a}\right)^{1/3}$

(D) $2a^{1/3}$

**25.** The value of $\sum_{r=1}^{5}\cos(2r-1)\frac{\pi}{11}$ is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

**26.** $\sin nx = \sum_{r=0}^{n} a_r \sin^r x$, where $n$ is an odd natural number. Then

(A) $a_0 = 1, a_1 = 2n$

(B) $a_0 = 1, a_1 = n$

(C) $a_0 = 0, a_1 = n$

(D) $a_0 = 0, a_1 = -n$

**27.** $\tan\frac{5\pi}{12} - \tan\frac{\pi}{12} - \sqrt{3}\tan\frac{5\pi}{12}\tan\frac{\pi}{12}$ is equal to

(A) $-\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) 1

(D) $\sqrt{3}$

**28.** The maximum value of $27^{\cos 2x} 81^{\sin 2x}$ is

**(A)** $3^2$ **(B)** $3^5$

**(C)** $3^7$ **(D)** 3

**29.** If $\cos\theta + \sin\theta = a$, $\cos 2\theta = b$, then

**(A)** $a^2 = b^2(2-a^2)$ **(B)** $b^2 = a^2(2-a^2)$

**(C)** $a^2 = b^2(2-b^2)$ **(D)** $b^2 = a^2(2-b^2)$

**30.** If angle $\theta$ is divided into two parts $A$ and $B$ such that $A - B = x$ and $\tan A : \tan B = k:1$, then the value of $\sin x$ is

**(A)** $\frac{k-1}{k+1}\sin\theta$ **(B)** $\frac{k+1}{k}\sin\theta$

**(C)** $\frac{k+1}{k-1}\sin\theta$ **(D)** None of these

**31.** If $\alpha$ and $\beta$ are solutions of $\sin^2 x + a\sin x + b = 0$ as well as of $\cos^2 x + c\cos x + d = 0$, then $\sin(\alpha + \beta)$ is equal to

**(A)** $\frac{2bd}{b^2+d^2}$ **(B)** $\frac{a^2+c^2}{2ac}$

**(C)** $\frac{b^2+d^2}{2bd}$ **(D)** $\frac{2ac}{a^2+c^2}$

**32.** If $\sin\theta$, $\cos\theta$ and $\tan\theta$ are in GP, then $\cot^6\theta - \cot^2\theta$ is equal to

**(A)** $\text{cosec}^2\theta$ **(B)** $\text{cosec}\,\theta$

**(C)** 1 **(D)** 0

**33.** If $3\cos x = 2\cos(x-2y)$, then $\tan(x-y)\tan y$ is equal to

**(A)** 5

**(B)** 6

**(C)** $\frac{1}{5}$

**(D)** $\frac{1}{6}$

**34.** If $A > 0$, $B > 0$ and $A+B=\frac{\pi}{3}$, then the maximum value of $\tan A \tan B$ is

**(A)** $\frac{1}{3}$

**(B)** 1

**(C)** $\sqrt{3}$

**(D)** $\frac{1}{\sqrt{3}}$

**35.** If in $\Delta ABC$, $\cos B + \cos C = 2 - 2\cos A$, then

**(A)** $a, b, c$ are in AP

**(B)** $b, a, c$ are in AP

**(C)** $c, a, b$ are in GP

**(D)** $c, b, a$ are in AP

**36.** The value of $\cos^2\frac{\pi}{9}+\cos^2\frac{2\pi}{9}+\cos^2\frac{4\pi}{9}$ is

**(A)** 0

**(B)** 3

**(C)** 9

**(D)** None of these

**37.** If $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$, then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is

**(A)** $\frac{1}{y}$

**(B)** $y$

**(C)** $1-y$

**(D)** $1+y$

**38.** If $4n\alpha = \pi$, then $\cot\alpha \cdot \cot 2\alpha \cdot \cot 3\alpha \cdots \cot(2n-1)\alpha$ is equal to

**(A)** 1
**(B)** $-1$
**(C)** $\infty$
**(D)** None of these

**39.** If $\sin\alpha + \sin\beta = a$ and $\cos\alpha - \cos\beta = b$, then $\tan\left(\frac{\alpha-\beta}{2}\right)$ is equal to

**(A)** $-\frac{a}{b}$
**(B)** $-\frac{b}{a}$
**(C)** $\sqrt{a^2+b^2}$
**(D)** None of these

**40.** If $\tan\theta = a \neq 0$, $\tan 2\theta = b \neq 0$ and $\tan\theta + \tan 2\theta = \tan 3\theta$, then

**(A)** $a = b$
**(B)** $ab = 1$
**(C)** $a + b = 0$
**(D)** $b = 2a$

**41.** The greatest real number among sin1, sin2, sin3 and $\sin(\sqrt{10}-2)$ is

**(A)** sin 1
**(B)** sin 2
**(C)** sin 3
**(D)** $\sin(\sqrt{10}-2)$

# -:SOLUTIONS:-

**Next Page**

**24.** $\cos^2\theta = \dfrac{a^2-1}{3}, \tan^2\dfrac{\theta}{2} = \tan^{2/3}\alpha$

Now,

$$\tan^3\frac{\theta}{2} = \tan\alpha \Rightarrow \frac{\sin^3(\theta/2)}{\cos^3(\theta/2)} = \frac{\sin\alpha}{\cos\alpha}$$

Let $\dfrac{\sin^3(\theta/2)}{\sin\alpha} = \dfrac{\cos^3(\theta/2)}{\cos\alpha} = k$. Then

$$\sin^3\frac{\theta}{2} = k\sin\alpha$$

$$\cos^3\frac{\theta}{2} = k\cos\alpha$$

Now,

$$k^{2/3}\sin^{2/3}\alpha + k^{2/3}\cos^{2/3}\alpha = 1$$

$$\Rightarrow \sin^{2/3}\alpha + \cos^{2/3}\alpha = \frac{1}{k^{2/3}}$$

Squaring and adding Eqs. (1) and (2), we get

$$k^2(\sin^2\alpha + \cos^2\alpha) = \sin^6\frac{\theta}{2} + \cos^6\frac{\theta}{2}$$

$$= \left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right)^3 - 3\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right)$$

$$\Rightarrow k^2 = 1 - \frac{3}{4}\sin^2\theta = 1 - \frac{3}{4} + \frac{3}{4}\cos^2\theta$$

$$\Rightarrow k^2 = \frac{a^2}{4} \Rightarrow k = \frac{a}{2}$$

Therefore,

$$\sin^{2/3}\alpha + \cos^{2/3}\alpha = \left(\frac{2}{a}\right)^{2/3}$$

**25.** $\sum_{r=1}^{5}\cos(2r-1)\frac{\pi}{11}$

$= \cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$

$=\frac{2\sin(\pi/11)}{2\sin(\pi/11)}\left(\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}\right)$

$$=\frac{\left(\sin\frac{2\pi}{11}+\sin\frac{4\pi}{11}-\sin\frac{2\pi}{11}+\sin\frac{6\pi}{11}-\sin\frac{4\pi}{11}+\sin\frac{8\pi}{11}-\sin\frac{6\pi}{11}+\sin\frac{10\pi}{11}-\sin\frac{8\pi}{11}\right)}{2\sin\frac{\pi}{11}}$$

$$=\frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}=\frac{\sin\left(\pi-\frac{\pi}{11}\right)}{2\sin\frac{\pi}{11}}=\frac{1}{2}$$

**26.** $\sin nx=\mathrm{Im}(e^{inx})=\mathrm{Im}\,[(\cos x+i\sin x)^n]$

$={}^nC_1\cos^{n-1}x\cdot\sin x-{}^nC_3\cos^{n-3}\sin^3 x+{}^nC_5\ \cos^{n-5}x\cdot\sin^5 x+\cdots$

Since $n$ is odd, let $n=2\lambda+1$. Then

$\sin nx={}^nC_1(\cos^2 x)^{\lambda}\sin x-{}^nC_3(\cos^2 x)^{\lambda-1}\sin^3 x+\cdots$

$={}^nC_1(1-\sin^2 x)^{\lambda}\sin x-{}^nC_3(1-\sin^2 x)^{\lambda-1}\cdot\sin^3 x+{}^nC_5(1-\sin^2 x)^{\lambda-2}\cdot\sin^5 x+\cdots$

$={}^nC_1\sin x-({}^nC_1\cdot{}^{\lambda}C_1+{}^nC_3)\sin^3 x+\cdots$

(1) $\Rightarrow a_0=0, a_1=n$

(2) **27.** We have $\tan\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)=\frac{\tan\frac{5\pi}{12}-\tan\frac{\pi}{12}}{1+\tan\frac{5\pi}{12}\cdot\tan\frac{\pi}{12}}$

$\Rightarrow\ \tan\frac{\pi}{3}=\frac{\tan\frac{5\pi}{12}-\tan\frac{\pi}{12}}{1+\tan\frac{5\pi}{12}\cdot\tan\frac{\pi}{12}}$

$\Rightarrow\ \tan\frac{5\pi}{12}-\tan\frac{\pi}{12}-\sqrt{3}\tan\frac{5\pi}{12}\cdot\tan\frac{\pi}{12}=\sqrt{3}$

**28.** $27^{\cos 2x}\cdot 81^{\sin 2x}=3^{3\cos 2x+4\sin 2x}$

Maximum value of $3\cos 2x+4\sin 2x=\sqrt{3^2+4^2}=5$

Therefore, maximum value of $27^{\cos 2x}\cdot 81^{\sin 2x}=3^5$

**29.** Given $\cos\theta+\sin\theta=a$

$\Rightarrow\ 1+\sin 2\theta=a^2$ [squaring both sides]

$\Rightarrow\ \sin 2\theta=a^2-1$

$\Rightarrow\ \cos^2 2\theta=1-(a^2-1)^2$

$\Rightarrow\ b^2=a^2(2-a^2)$

**30.** Given $\frac{\tan A}{\tan B}=\frac{k}{1}$

$\Rightarrow\frac{\sin A}{\cos A}\cdot\frac{\cos B}{\sin B}=\frac{k}{1}$

$\Rightarrow\ \frac{\sin(A-B)}{\sin(A+B)}=\frac{k-1}{k+1}$ [By Componendo and Dividendo]

$\Rightarrow\ \sin x=\frac{k-1}{k+1}\sin\theta$ [$\because A-B=x$ and $A+B=\theta$]

**31.** Given $\sin\alpha+\sin\beta=-a$, $\sin\alpha\sin\beta=b$

$\cos\alpha+\cos\beta=-c$, $\cos\alpha\cos\beta=d$

So, $2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}=-a$

and $2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}=-c$

Therefore, $\tan\frac{\alpha+\beta}{2}=\frac{a}{c}$

$\Rightarrow\sin(\alpha+\beta)=\frac{2a/c}{1+a^2/c^2}=\frac{2ac}{a^2+c^2}$

**32.** Given $\sin\theta$, $\cos\theta$ and $\tan\theta$ are in GP. This means

$\cos^2\theta=\frac{\sin^2\theta}{\cos\theta}\quad\Rightarrow\ \sin^2\theta=\cos^3\theta\Rightarrow\operatorname{cosec}\theta=\cot^3\theta$

Now,

$$\cot^6\theta - \cot^2\theta = \text{cosec}^2\theta - \cot^2\theta = 1$$

**33.** Given $\dfrac{\cos(x-2y)}{\cos x} = \dfrac{3}{2}$

$\Rightarrow \dfrac{\cos(x-2y)-\cos x}{\cos(x-2y)+\cos x} = \dfrac{1}{5}$ [By Componendo and Dividendo]

$\Rightarrow \dfrac{2\sin(x-y)\cdot\sin y}{2\cos(x-y)\cdot\cos y} = \dfrac{1}{5}$

$\Rightarrow \tan(x-y)\cdot\tan y = \dfrac{1}{5}$

**34.** Given $A+B = \dfrac{\pi}{3} \;\Rightarrow\; B = \dfrac{\pi}{3} - A$

Let $k = \tan A \tan B$

$= \tan A\cdot\tan\left(\dfrac{\pi}{3} - A\right)$

$= \tan A\cdot\dfrac{\sqrt{3}-\tan A}{1+\sqrt{3}\tan A}$

$\Rightarrow \tan^2 A + \sqrt{3}(k-1)\tan A + k = 0$

Since tan $A$ is real,

$3(k-1)^2 - 4k \ge 0$

$\Rightarrow (3k-1)(k-3) \ge 0$

$\Rightarrow k \le \dfrac{1}{3}$ or $k \ge 3$,

but $k$ cannot be greater than 3, since $A+B = \pi/3$.

Therefore, maximum value of tan $A$ tan $B$ is 1/3.

**35.** Given $\cos B + \cos C = 2 - 2\cos A$

$\Rightarrow 2\cos\dfrac{B+C}{2}\cdot\cos\dfrac{B-C}{2} = 4\sin^2\dfrac{A}{2}$

$\Rightarrow \cos\dfrac{B-C}{2} = 2\sin\dfrac{A}{2}$

$\Rightarrow \sin\dfrac{B+C}{2}\cdot\cos\dfrac{B-C}{2} = \sin A$

$\Rightarrow \sin B + \sin C = 2\sin A$

$\Rightarrow$ sin $B$, sin $A$, sin $C$ are in AP

$\Rightarrow$ $B$, $A$, $C$ are in AP

**36.** We have $\cos^2\dfrac{\pi}{9} + \cos^2\dfrac{2\pi}{9} + \cos^2\dfrac{4\pi}{9}$

$= \dfrac{1}{2}\left[1+\cos\dfrac{2\pi}{9} + 1 + \cos\dfrac{4\pi}{9} + 1 + \cos\dfrac{8\pi}{9}\right]$

$= \dfrac{1}{2}\left[3 + 2\cos\dfrac{\pi}{3}\cdot\cos\dfrac{\pi}{9} + \cos\left(\pi - \dfrac{\pi}{9}\right)\right]$

$= \dfrac{1}{2}\left[3 + \cos\dfrac{\pi}{9} - \cos\dfrac{\pi}{9}\right]$

$= \dfrac{3}{2}$

**37.** Given $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

$$\Rightarrow \frac{4\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = y$$

$$\Rightarrow \frac{2\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}} = y$$

Now,

$$\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \frac{2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}{\left(\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}\right)^2}$$

$$= \frac{2\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \sin\frac{\alpha}{2}} = y$$

**38.** Given $4n\alpha = \pi$

Now, $\cot\alpha\cdot\cot 2\alpha\cdot\cot 3\alpha\cdots\cot(2n-3)\alpha\cdot\cot(2n-2)\alpha\cdot\cot(2n-1)\alpha$

$= \cot\alpha\cdot\cot 2\alpha\cdot\cot 3\alpha\cdots\cot\left(\dfrac{\pi}{2} - 3\alpha\right)\cdot\cot\left(\dfrac{\pi}{2} - 2\alpha\right)\cdot\cot\left(\dfrac{\pi}{2} - \alpha\right)$

$= \cot\alpha\cdot\cot 2\alpha\cdot\cot 3\alpha\cdots\tan 3\alpha\cdot\tan 2\alpha\cdot\tan\alpha = 1$

**39.** Given $\sin\alpha + \sin\beta = a$. Now

$$2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2} = a$$

and $\cos\alpha - \cos\beta = b$

So we have

$$-2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2} = b$$

Therefore, $\tan\dfrac{\alpha-\beta}{2} = -\dfrac{b}{a}$

**40.** Given $\tan\theta + \tan 2\theta = \tan 3\theta$

$\Rightarrow \tan\theta + \tan 2\theta = \dfrac{\tan\theta + \tan 2\theta}{1 - \tan\theta\cdot\tan 2\theta}$

$\Rightarrow (\tan\theta + \tan 2\theta)\left(1 - \dfrac{1}{1-\tan\theta\tan 2\theta}\right) = 0$

$\Rightarrow \tan\theta + \tan 2\theta = 0$ or $\tan\theta\tan 2\theta = 0$

$\Rightarrow a + b = 0$ $[\because ab \ne 0]$

**41.** We know that $\sin x$ is an increasing function in $\left[0, \frac{\pi}{2}\right]$

Now,

$\sin 2 = \sin(\pi - 2) = \sin 1.14$

$\sin 3 = \sin(\pi - 3) = \sin 0.14$

$\sin(\sqrt{10} - 2) = \sin 1.16$

Therefore, among $\sin 1$, $\sin 2$, $\sin 3$ and $\sin(\sqrt{10} - 2)$, $\sin(\sqrt{10} - 2)$ is greatest.

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