ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-5

SUBJECT – MATHEMATICS

Pre-test

Chapter: MATRICES AND DETERMINANTS **Class: XII**

Topic: DETERMINANTS **Date: 13.05.2020**

PART 2

SOLVED PROBLEMS

1. Let λ and α be real. The set of all values of λ for which the system of linear equations

$$\lambda x + (\sin\alpha)y + (\cos\alpha)z = 0$$
$$x + (\sin\alpha)y - (\cos\alpha)z = 0$$
$$-x + (\sin\alpha)y + (\cos\alpha)z = 0$$

has a non-trivial solution is

(a) $[0, \sqrt{2}]$ (b) $[-\sqrt{2}, 0]$

(c) $[-\sqrt{2}, \sqrt{2}]$ (d) none of these

Ans. (c)

Solution: Since the system has a non-trivial solution, therefore

$$\begin{vmatrix} \lambda & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ -1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$\Rightarrow \lambda(-\cos^2\alpha - \sin^2\alpha) - (-\sin\alpha\cos\alpha - \sin\alpha\cos\alpha)$
$\quad - (\sin^2\alpha - \cos^2\alpha) = 0$

$\Rightarrow -\lambda + \sin 2\alpha + \cos 2\alpha = 0 \Rightarrow \lambda = \sin 2\alpha + \cos 2\alpha$

$\Rightarrow \lambda = \sqrt{2}\cos\left(2\alpha - \frac{\pi}{4}\right)$

Since $-1 \le \cos\left(2\alpha - \frac{\pi}{4}\right) \le 1 \, \forall \alpha \in R$

$\therefore -\sqrt{2} \le \lambda \le \sqrt{2}$ i.e. $\lambda \in [-\sqrt{2}, \sqrt{2}]$

2. The value of the determinant $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix}$ is

(a) $abc + pqr + xyz$ (b) $(a-x)(y-z)(r-p)$

(c) 0 (d) none of these

Ans. (c)

Solution: Applying $C_1 \to C_1 + C_2 + C_3$, we get the value of the determinant = 0.

3. Which of the following is not the root of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$?

(a) -9 (b) 2

(c) 3 (d) 7

Ans. (c)

Solution: $\Delta = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$

$$\Delta = \begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} (R_1 \to R_1 + R_2 + R_3)$$

$$= \begin{vmatrix} x+9 & 0 & 0 \\ 2 & x-2 & 0 \\ 7 & -1 & x-7 \end{vmatrix} (C_2 \to C_2 - C_1, C_3 \to C_3 - C_1)$$

$= (x + 9)(x - 2)(x - 7)$

$\therefore \quad \Delta = 0$

$\Rightarrow \quad (x + 9)(x - 2)(x - 7) = 0$

$\Rightarrow \quad x = -9, 2, 7.$

4. If $a + b + c = 0$, one root of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ is

(a) $x = 0$ (b) $x = 1$

(c) $x = 2$ (d) $x = a^2 + b^2 + c^2$

Ans. (a)

Solution: Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$(a+b+c-x)\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$$

i.e. $$x\begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix} = 0$$

$\therefore x = 0$ is one of the roots of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0.$

5. If $\Delta = \begin{vmatrix} a & c & c-a & a+c \\ c & b & b-c & b+c \\ a-b & b-c & 0 & a-c \\ x & y & z & 1+x+y \end{vmatrix} = 0$ and a, b, c are not in A.P., then

(a) a, c, b are in H.P. (b) a, c, b are in G.P.
(c) b, a, c in A.P. (d) a, b, c are in G.P.

Ans. (b)

Solution: Applying $C_3 \to C_3 - (C_2 - C_1)$ and $C_4 \to C_4 - (C_1 - C_2)$, we get

$$\Delta = \begin{vmatrix} a & c & 0 & 0 \\ c & b & 0 & 0 \\ a-b & b-c & a+c-2b & 0 \\ x & y & z+x-y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a & c & 0 \\ c & b & 0 \\ a-b & b-c & a+c-2b \end{vmatrix} \quad \text{(Expanding along } C_4)$$

$$= (a+c-2b)(ab-c^2) \quad \text{(Expanding along } C_3)$$

$\therefore \quad \Delta = 0$

$\Rightarrow \quad a+c-2b = 0$ or $ab - c^2 = 0$.

$\Rightarrow$ a, c, b are in G.P. (given that a, b, c are in A.P.)

6. The value of $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ is

(a) abc (b) 0
(c) $a + b + c$ (d) $4abc$

Ans. (d)

Solution: We have $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

(Applying $R_1 \to R_1 - R_2 - R_3$)

$$= \frac{1}{c}\begin{vmatrix} 0 & -2c & -2b \\ 0 & c(c+a-b) & b(c-a-b) \\ c & c & a+b \end{vmatrix}$$

(Applying $R_2 \to cR_2 - bR_3$)

$$= \frac{1}{c} \cdot c(2bc)\left[-(c-a-b)+(c+a-b)\right]$$

$$= (2bc)(2a) = 4abc \quad \text{(Expanding along } C_1)$$

7. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is equal to

(a) 1 (b) 0
(c) $(1-a)(1-b)(1-c)$ (d) $(a+b+c)$

Ans. (b)

Solution: Let $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

$$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} (C_3 \to C_2 + C_3)$$

$$= (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} = 0 \quad [\because \text{two columns are identical}]$$

8. The value of $\begin{vmatrix} yz & zx & xy \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$, where x, y, z are, respectively, the pth, $(2q)$th and $(3r)$th terms of an H.P., is

(a) -1 (b) 0
(c) 1 (d) none of these

Ans. (b)

Solution: Let a be the first term and d be the common difference of corresponding A.P.

Then, $$\Delta = xyz\begin{vmatrix} 1/x & 1/y & 1/z \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= xyz\begin{vmatrix} a+(p-1)d & a+(2q-1)d & a+(3r-1)d \\ p & 2q & 3r \\ 1 & 1 & 1 \end{vmatrix}$$

Applying $R_1 \to R_1 - aR_3, R_2 \to R_2 - R_3$ and then taking d common from R_1, we get

$$\Delta = xyzd\begin{vmatrix} (p-1) & (2q-1) & (3r-1) \\ (p-1) & (2q-1) & (3r-1) \\ 1 & 1 & 1 \end{vmatrix} = 0$$

9. The value of the determinant $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$ is

(a) -2 (b) $x^2 + 2$
(c) 2 (d) none of these

Ans. (a)

Solution: We have

$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} = \begin{vmatrix} x+1 & x+2 & 2 \\ x+3 & x+5 & 3 \\ x+7 & x+10 & 4 \end{vmatrix}$$

(Applying $C_3 \to C_3 - C_2$)

$$= \begin{vmatrix} x+1 & 1 & 2 \\ x+3 & 2 & 3 \\ x+7 & 3 & 4 \end{vmatrix} \quad \text{(Applying } C_2 \to C_2 - C_1\text{)}$$

$$= \begin{vmatrix} x+1 & 1 & 2 \\ 2 & 1 & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

(Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_2$)

$$= \begin{vmatrix} x+1 & 1 & 1 \\ 2 & 1 & 0 \\ 2 & 0 & 0 \end{vmatrix} \quad \text{(Applying, } R_3 \to R_3 - R_2\text{)}$$

$$= 2(0-1) = -2 \quad \text{(Expanding along } R_3\text{)}$$

10. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$ is equal to

(a) $abc(a+b+c)^2$ (b) $2abc(a+b+c)^2$

(c) $2abc(a+b+c)^3$ (d) $2abc(a+b+c)$

Ans. (c)

Solution: Let $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

(Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$)

$$= \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ 0 & (c+a)^2 - b^2 & b^2 \\ c^2 - (a+b)^2 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (c+a+b)(c+a-b) & b^2 \\ (c+a+b)(c-a-b) & (c+a+b)(c-a-b) & (a+b)^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

[Applying $R_3 \to R_3 - (R_1 + R_2)$]

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac & a^2 & a^2 \\ b^2 & bc+ba & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$

$$= \frac{(a+b+c)^2}{ab} \cdot ab \cdot 2ab \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 2ab(a+b+c)^2 \begin{vmatrix} b+c & a \\ b & c+a \end{vmatrix}$$

$$= 2ab(a+b+c)^2 [(b+c)(c+a) - ab]$$

$$= 2abc(a+b+c)^2 (a+b+c)$$

$$= 2abc(a+b+c)^3$$

11. If $g(x) = \begin{vmatrix} a^{-x} & e^{x \log_e a} & x^2 \\ a^{-3x} & e^{3x \log_e a} & x^4 \\ a^{-5x} & e^{5x \log_e a} & 1 \end{vmatrix}$, then

(a) $g(x) + g(-x) = 0$ (b) $g(x) - g(-x) = 0$

(c) $g(x) \times g(-x) = 0$ (d) none of these

Ans. (a)

Solution: $g(x) = \begin{vmatrix} a^{-x} & e^{\log_e a^x} & x^2 \\ a^{-3x} & e^{\log_e a^{3x}} & x^4 \\ a^{-5x} & e^{\log_e a^{5x}} & 1 \end{vmatrix}$

$$= \begin{vmatrix} a^{-x} & a^x & x^2 \\ a^{-3x} & a^{3x} & x^4 \\ a^{-5x} & a^{5x} & 1 \end{vmatrix} \quad [\because e^{\log a^x} = a^x]$$

$$\Rightarrow g(-x) = \begin{vmatrix} a^x & a^{-x} & x^2 \\ a^{3x} & a^{-3x} & x^4 \\ a^{5x} & a^{-5x} & 1 \end{vmatrix} = -\begin{vmatrix} a^{-x} & a^x & x^2 \\ a^{-3x} & a^{3x} & x^4 \\ a^{-5x} & a^{5x} & 1 \end{vmatrix}$$

(Interchanging 1st and 2nd columns)

$$= -g(x)$$

$$\Rightarrow g(x) + g(-x) = 0$$

12. If $f(x) = \begin{vmatrix} \cos^2 x & \cos x \cdot \sin x & -\sin x \\ \cos x \cdot \sin x & \sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then for all x

(a) $f(x) = 0$ (b) $f(x) = 1$

(c) $f(x) = 2$ (d) none of these

Ans. (b)

Solution: We have $f(x) = \begin{vmatrix} \cos^2 x & \cos x \cdot \sin x & -\sin x \\ \cos x \cdot \sin x & \sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 0 & -\sin x \\ 0 & 1 & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$$

(Applying $C_1 \to C_1 - \sin x \cdot C_3$ and $C_2 \to C_2 + \cos x \cdot C_3$)

$$= \begin{vmatrix} 1 & 0 & -\sin x \\ 0 & 1 & \cos x \\ 0 & -\cos x & \sin^2 x \end{vmatrix}$$

(Applying $R_3 \to R_3 - \sin x \cdot R_1$)

$= \sin^2 x + \cos^2 x = 1$ for all x (Expanding along C_1)

13. The value of the determinant $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ is

(a) $a^3 + b^3 + c^3 - 3abc$ (b) $-(a^3 + b^3 + c^3 - 3abc)$

(c) $-(a^3 + b^3 + c^3)$ (d) none of these

Ans. (b)

Solution: $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = \begin{vmatrix} a+b+c & -b & a \\ a+b+c & -c & b \\ a+b+c & -a & c \end{vmatrix}$

$[C_1 \to C_1 + C_3$ and $C_2 \to C_2 - C_3]$

$$= -(a+b+c) \begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & a & c \end{vmatrix}$$

$$= -(a+b+c) \begin{vmatrix} 1 & b & a \\ 0 & c-b & b-a \\ 0 & a-b & c-a \end{vmatrix}$$

$[R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1]$

$= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= -(a^3 + b^3 + c^3 - 3abc)$

14. If $a + b + c \neq 0$ and $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$, then the total number of different values of x is equal to

(a) 1 (b) 2

(c) 3 (d) none of these

Ans. (c)

Solution: Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} a+b+c-x & c & b \\ a+b+c-x & b-x & a \\ a+b+c-x & a & c-x \end{vmatrix}$$

$$= (a+b+c-x) \begin{vmatrix} 1 & c & b \\ 1 & b-x & a \\ 1 & a & c-x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$, we get

$$\Delta = (a+b+c-x) \begin{vmatrix} 0 & c-b+x & b-a \\ 0 & b-a-x & a-c+x \\ 1 & a & c-x \end{vmatrix}$$

$= (a + b + c - x) [(x + c - b)(x + a - c)$
$+ (x + a - b)(b - a)]$

$= (a+b+c-x)[x^2 + x(a-b) + (c-b)(a-c)$
$+ x(b - a) - (a - b)^2$

$= (a+b+c-x)(x^2 + ac - c^2 - ab + bc - a^2 - b^2 - 2ab)$

$= (a+b+c-x)(x^2 - a^2 - b^2 - c^2 + ab + bc + ca)$

$= (a+b+c-x)\left\{x^2 - \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]\right\}$

As $\Delta = 0$

$\Rightarrow x = a+b+c, \pm\sqrt{\frac{1}{2}((a-b)^2 + (b-c)^2 + (c-a)^2)}$

15. If A, B and C are the angles of a triangle, and

$$\Delta = \begin{vmatrix} e^{-2iA} & e^{iC} & e^{iB} \\ e^{iC} & e^{-2iB} & e^{iA} \\ e^{iB} & e^{iA} & e^{-2iC} \end{vmatrix}$$

find $|\Delta|$.

(a) 2 (b) 3

(c) -1 (d) -4

Ans. (d)

Solution: Let $\alpha = e^{iA}$, $\beta = e^{iB}$ and $\gamma = e^{iC}$, then $\alpha\beta\gamma = e^{i(A+B+C)}$ $= e^{i\pi} = \cos\pi + i\sin\pi = -1$ and

$$\Delta = \begin{vmatrix} 1/\alpha^2 & \gamma & \beta \\ \gamma & 1/\beta^2 & \alpha \\ \beta & \alpha & 1/\gamma^2 \end{vmatrix}$$

$$= \frac{1}{\alpha^2\beta^2\gamma^2} \begin{vmatrix} 1 & \alpha^2\gamma & \alpha^2\beta \\ \beta^2\gamma & 1 & \beta^2\alpha \\ \gamma^2\beta & \gamma^2\alpha & 1 \end{vmatrix} = \begin{vmatrix} 1 & -\alpha/\beta & -\alpha/\gamma \\ -\beta/\alpha & 1 & -\beta/\gamma \\ -\gamma/\alpha & -\gamma/\beta & 1 \end{vmatrix}$$

$$= \frac{1}{\alpha\beta\gamma} \begin{vmatrix} \alpha & -\alpha & -\alpha \\ -\beta & \beta & -\beta \\ -\gamma & -\gamma & \gamma \end{vmatrix}$$

[Multiplying C_1, C_2, C_3 by α, β and γ respectively]

$$= \frac{\alpha\beta\gamma}{\alpha\beta\gamma}\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & -1 \\ 0 & 1 & -1 \\ -2 & -1 & 1 \end{vmatrix} = -2(1+1) = -4$$

(Applying $C_1 \to C_1 + C_2$)

16. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is equal to

(a) abc (b) $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(c) $abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ (d) $abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

Ans. (d)

Solution: Let $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$

$$= abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$= abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$(R_1 \to R_1 + R_2 + R_3)$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

$(C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1)$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

17. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$ is equal to

(a) -1 (b) 1
(c) -2 (d) 2

Ans. (d)

Solution: Given $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$ reduces the determinant to

$$\begin{vmatrix} p-a & b-q & 0 \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0$$

$\Rightarrow$ $(p-a)(q-b)r + a(b-q)(c-r) - b(p-a)(c-r) = 0$

$\Rightarrow$ Dividing throughout by $(p-a)(q-b)(r-c)$, we get

$$\frac{r}{r-c}+\frac{a}{p-a}+\frac{b}{q-b} = 0$$

$$\Rightarrow \quad \frac{r}{r-c}+1+\frac{a}{p-a}+1+\frac{b}{q-b} = 2$$

$$\Rightarrow \quad \frac{r}{r-c}+\frac{p}{p-a}+\frac{q}{q-b} = 2$$

18. If $U_n = \begin{vmatrix} 1 & k & k \\ 2n & k^2+k+1 & k^2+k \\ 2n-1 & k^2 & k^2+k+1 \end{vmatrix}$ and $\sum_{n=1}^{k} U_n = 72$

then $k =$

(a) 8 (b) 9
(c) 6 (d) none of these

Ans. (a)

Solution: $\sum_{n=1}^{k} U_n = \begin{vmatrix} \sum_{n=1}^{k} 1 & k & k \\ 2\sum_{n=1}^{k} n & k^2+k+1 & k^2+k \\ 2\sum_{n=1}^{k} n - \sum_{n=1}^{k} 1 & k^2 & k^2+k+1 \end{vmatrix}$

$$= \begin{vmatrix} k & k & k \\ k(k+1) & k^2+k+1 & k^2+k \\ k^2 & k^2 & k^2+k+1 \end{vmatrix}$$

$$= \begin{vmatrix} k & 0 & k \\ k^2+1 & 1 & k^2+k \\ k^2 & 0 & k^2+k+1 \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$]

$$= k(k^2+k+1) - k^3 = k(k+1) = 72$$

$\Rightarrow \quad k = 8.$

19. If $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$ and a, b, c are all distinct, then $abc(ab+bc+ca)$ is equal to

(a) $a+b+c$ (b) abc
(c) 0 (d) none of these

Ans. (a)

Solution: Given $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} + \begin{vmatrix} a & a^3 & -1 \\ b & b^3 & -1 \\ c & c^3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow abc\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} - 1\begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow abc(a-b)(b-c)(c-a)(ab+bc+ca) - (a-b)(b-c)(c-a)(a+b+c) = 0$$

$$\Rightarrow abc(ab+bc+ca) - (a+b+c) = 0$$

$$\Rightarrow abc(ab+bc+ca) = a+b+c$$

EXERCISE SET 1

1. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$,

then $f(500)$ is equal to

(a) 0 (b) 1
(c) 500 (d) -500

2. Let $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then

$f\left(\frac{\pi}{6}\right) = ?$

(a) 1 (b) 0
(c) -1 (d) 2

3. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2+bx+c=0$ and $px^2+qx+r=0$, respectively. If the system of equations $a_1y+a_2z=0$ and $\beta_1y+\beta_2z=0$ has a non-trivial solution, then

(a) $\frac{b^2}{q^2} = \frac{ac}{pr}$ (b) $\frac{c^2}{r^2} = \frac{ab}{pq}$
(c) $\frac{a^2}{p^2} = \frac{bc}{qr}$ (d) none of these

4. A root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is

(a) 6 (b) 3
(c) 0 (d) none of these

5. If $l_1^2+m_1^2+n_1^2 = 1$, etc., and $l_1l_2+m_1m_2+n_1n_2 = 0$, etc.,

and $\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$, then

(a) $|\Delta| = 3$ (b) $|\Delta| = 2$
(c) $|\Delta| = 1$ (d) $\Delta = 0$

6. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, ω being a cube root of unity, is

(a) 0 (b) 1
(c) ω^2 (d) ω

7. The value of the determinant

$$\begin{vmatrix} 1 & \sin 3\theta & \sin^3\theta \\ 2\cos\theta & \sin 6\theta & \sin^3 2\theta \\ 4\cos^2\theta - 1 & \sin 9\theta & \sin^3 3\theta \end{vmatrix}$$ is

(a) -1 (b) 0
(c) 1 (d) 2

8. $\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = ?$

(a) abc (b) $a+b+c$
(c) 0 (d) $a^2+b^2+c^2$

9. If $a \neq b \neq c$, one value of x which satisfies the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$ is given by

(a) $x = a$ (b) $x = b$
(c) $x = c$ (d) $x = 0$

10. If a, b, c are non-zero real numbers, then $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix}$ vanishes for

(a) $\frac{1}{a}-\frac{1}{b}-\frac{1}{c}=0$ (b) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$
(c) $\frac{1}{b}+\frac{1}{c}-\frac{1}{a}=0$ (d) $\frac{1}{b}-\frac{1}{c}-\frac{1}{a}=0$

11. If $A + B + C = \pi$, then the value of

$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$ is equal to

(a) 0 (b) 1
(c) $2 \sin B \tan A \cos C$ (d) none of these

12. $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix} = ?$

(a) $(x+p)(x+q)(x-p-q)$
(b) $(x-p)(x-q)(x+p+q)$
(c) $(x-p)(x-q)(x-p-q)$
(d) $(x+p)(x+q)(x+p+q)$

13. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ if

(a) a, b, c are in A.P.
(b) a, b, c are in G.P. or $(x-\alpha)$ is a factor of $ax^2+2bx+c=0$
(c) a, b, c are in H.P.
(d) α is a root of the equation $ax^2 + 2bx + c = 0$

14. $f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then $f\left(\frac{\pi}{3}\right)$ is equal to

(a) -5 (b) -4
(c) -3 (d) -1

15. For a +ve value of x, y, z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is equal to

(a) 1 (b) 0
(c) $\log x + \log y + \log z$ (d) -1

16. The value of the determinant $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$ is

(a) $kabc(a^2+b^2+c^2)$
(b) $k(a+b)(b+c)(c+a)$
(c) $k(a-b)(b-c)(c-a)$
(d) $k(a+b-c)(b+c-a)(c+a-b)$

17. If $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 56$, then n equals

(a) 4 (b) 6
(c) 7 (d) none of these

18. $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2$ is equal to

(a) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

(b) $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & bc & a^2+b^2 \end{vmatrix}$

(c) $\begin{vmatrix} a+b & 0 & c \\ b+c & b & 0 \\ c+a & a & b \end{vmatrix}$

(d) $\begin{vmatrix} b+c & ab & ac \\ c+a & ca & cb \\ a+b & ba & bc \end{vmatrix}$

19. If $a+b+c=0$, then $\Delta = \begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ when

(a) $x = -1$ (b) $x = \pm\sqrt{\frac{3}{2}(a^2+b^2+c^2)}$
(c) $x = abc$ (d) none of these

20. The number of distinct real solutions of

$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

(a) 0 (b) 2
(c) 1 (d) 3

21. If $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$, then $x = ?$

(a) 1, 9 (b) –1, 9
(c) –1, –9 (d) 1, –9

22. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ if

(a) x, y, z are in A.P. (b) x, y, z are in G.P.
(c) x, y, z are in H.P. (d) xy, yz, zx are in A.P.

23. The value of $\Delta = \begin{vmatrix} ^{10}C_4 & ^{10}C_5 & ^{11}C_m \\ ^{11}C_6 & ^{11}C_7 & ^{12}C_{m+2} \\ ^{12}C_8 & ^{12}C_9 & ^{13}C_{m+4} \end{vmatrix}$ is equal to zero, where m is

(a) 6 (b) 4
(c) 5 (d) none of these

24. Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$, then the maximum value of $f(x)$ is

(a) 0 (b) 1
(c) 6 (d) –1

25. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ 1 & 1 & 1 \\ \beta & \gamma & \alpha \end{vmatrix}$ is

(a) $p^2 + q - 2r$ (b) $p^2 - 3q$
(c) $p^2 + q + 2r$ (d) $p^2 + 3q$

26. The value of the determinant $\begin{vmatrix} ^5C_0 & ^5C_3 & 1 \\ ^5C_1 & ^5C_4 & 1 \\ ^5C_2 & ^5C_5 & 1 \end{vmatrix}$ is

(a) 0 (b) –(6!)
(c) 80 (d) none of these

27. The value of the determinant $\begin{vmatrix} a+pd & a+qd & a+rd \\ p & q & r \\ d & d & d \end{vmatrix}$ is equal to

(a) 0 (b) –1
(c) 1 (d) $p+q+r$

28. The value of $\Delta = \begin{vmatrix} ^5C_0 & ^5C_3 & 14 \\ ^5C_1 & ^5C_4 & 1 \\ ^5C_2 & ^5C_5 & 1 \end{vmatrix}$ is

(a) 0 (b) –576
(c) 80 (d) none of these

29. If $f(x)$, $g(x)$ and $h(x)$ are three polynomials of degree 2 and $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ then $\Delta(x)$ is a polynomial of degree

(a) 2 (b) 3
(c) 0 (d) at most 3

30. The value of $\begin{vmatrix} 2 & \omega & \omega^2 \\ 1 & 1+\omega^2 & \omega \\ \omega^2 & 1 & 1+\omega \end{vmatrix}$, where ω is the complex cube root of unity, is

(a) 1 (b) ω
(c) ω^2 (d) none of these

31. Let $f(x) = \begin{vmatrix} \cos x & -x & 1 \\ 2\sin x & -x^2 & 2x \\ \tan x & -x & 1 \end{vmatrix}$, find $\lim_{x \to 0} \frac{f'(x)}{x}$.

(a) 3 (b) 4
(c) 2 (d) 3

32. If a, b, c are in A.P., then the determinant $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ in its simplified form is

(a) $x^3 + 3ax + 7c$ (b) 0
(c) 15 (d) $10x^2 + 5x + 2c$

33. If $\Delta(x) = \begin{vmatrix} x & 1+x^2 & x^3 \\ \log(1+x^2) & e^x & \sin x \\ \cos x & \tan x & \sin^2 x \end{vmatrix}$, then

(a) $\Delta(x)$ is divisible by x (b) $\Delta(x) = 0$
(c) $\Delta'(x) = 0$ (d) none of these

34. If $\Delta(n) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ \alpha & \alpha^2 & \alpha^3 \end{vmatrix}$, then the value of $\frac{d^n}{dx^n}[\Delta(x)]$ at $x = 0$ is
(a) -1 (b) 0
(c) 1 (d) 2

35. If $\begin{vmatrix} 3x^2-2x+3 & x^2-5 & 3x-2 \\ x & 2-3x & x+1 \\ x^2+2x & 7+2x^2 & 3x-x^2+1 \end{vmatrix} = ax^5+bx^4+cx^3$ $+ dx^3 + ex + f$, then f is equal to
(a) -12 (b) -15
(c) 15 (d) 12

36. A root of the equation $\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$ is
(a) a (b) b
(c) 0 (d) 1

37. If $2s = a+b+c$, then the value of $\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}$ is
(a) $\sqrt{s(s-a)(s-b)(s-c)}$
(b) $2s^2(s-a)(s-b)(s-c)$
(c) $2s^3(s-a)(s-b)(s-c)$
(d) none of these

38. If α, β, γ are the roots of $px^3 + qx^2 + r = 0$, then the value of the determinant $\begin{vmatrix} \alpha\beta & \beta\gamma & \gamma\alpha \\ \beta\gamma & \gamma\alpha & \alpha\beta \\ \gamma\alpha & \alpha\beta & \beta\gamma \end{vmatrix}$ is
(a) p (b) q
(c) 0 (d) r

39. If $f(x) = \begin{vmatrix} 1 & 2x & 3x^2 \\ 3 & a & 27 \\ 1 & 3 & 9 \end{vmatrix}$ and $\int_0^3 f(x)dx = 0$, then a is equal to
(a) 3 (b) 6
(c) 9 (d) any real number

40. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5+bx^4+cx^3+dx^2+\lambda x+\mu$ be an identity in x, where a, b, c, d, λ, μ are independent of x. Then the value of λ is
(a) 3 (b) 2
(c) 4 (d) none of these

41. If $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \lambda a^2b^2c^2$, then the value of λ is
(a) 1 (b) 2
(c) 4 (d) 3

42. The value of the determinant $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$ is
(a) $a^3\left(1-\frac{2}{a}\right)$ (b) $a^3\left(1+\frac{3}{a}\right)$
(c) $a^3\left(1-\frac{3}{a}\right)$ (d) $a^3\left(1+\frac{2}{a}\right)$

43. If $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = 64$, then $a = ?$
(a) 2 (b) 3
(c) 4 (d) 6

44. For positive numbers x, y, z, the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_z x & 2 & \log_y z \\ \log_z x & \log_z y & 4 \end{vmatrix}$ equals
(a) 0 (b) 3
(c) $\log x \log y \log z$ (d) none of these

45. If a, b, c are the sides of a triangle and $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$, then
(a) ΔABC is an equilateral triangle
(b) ΔABC is a right-angled isosceles triangle
(c) ΔABC is an isosceles triangle
(d) none of these

46. If $D_r = \begin{vmatrix} 2^{r-1} & 2\cdot 3^{r-1} & 4\cdot 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$, then the value of $\sum_{r=1}^{n} D_r$ is

(a) 0 (b) $\alpha\beta\gamma$
(c) $\alpha+\beta+\gamma$ (d) $\alpha\cdot 2^n+\beta\cdot 3^n+\gamma\cdot 4^n$

47. If a, b, c are non-zero distinct numbers, then the system of equations $ax + by + cz = 0$, $cx + ay + bz = 0$ and $bx + cy + ax = 0$ has a non-trivial solution provided
(a) $a+b+c=0$ (b) $a+b+c\neq 0$
(c) $a+b+c=1$ (d) $a+b+c\ -1$

48. If $f(x)=\begin{vmatrix} x^3 & \cos^2 x & 2^{x^4} \\ \tan^5 x & 1 & \sec 2x \\ \sin^3 x & x^4 & 5 \end{vmatrix}$, then $\int_{-\pi/2}^{\pi/2} f(x)dx=?$
(a) 2 (b) −2
(c) 0 (d) none of these

49. If $f(x)=\begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin(2x) & 0 & \sin(2x^2) \end{vmatrix}$, find $f'(0)$.
(a) 2 (b) 4
(c) 7 (d) 1

50. The equations $ax+y=4$, $4x+ay=8$ and $2x+y=2a$ are consistent for
(a) no value of a (b) one value of a
(c) two values of a (d) three values of a

51. If $f(x)=\begin{vmatrix} \sec^2 x & 1 & 1 \\ \cos^2 x & \cos^2 x & \text{cosec}^2 x \\ 1 & \cos^2 x & \cot^2 x \end{vmatrix}$, then $f(x)=?$
(a) $\cos^4 x$ (b) $\sin^4 x$
(c) 0 (d) $\sin^2 x\cos^2 x$

52. If $S_r=\begin{vmatrix} 2r & x & n(n+1) \\ 6r^2-1 & y & n^2(2n+3) \\ 4r^3-2nr & z & n^3(n+1) \end{vmatrix}$, then the value of $\sum_{r=1}^{n} S_r$ is independent of
(a) x only (b) y only
(c) x, y, z, n (d) n only

53. If the system of equations $ax + y + z = 0$, $x + by + z = 0$ and $x + y + cz = 0$ ($a, b, c \neq 1$) has a non-trivial solution, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is
(a) −1 (b) 0
(c) 1 (d) none of these

54. The system of equations $3x+ky=2, kx+ky=4$ is inconsistent for
(a) no value of k (b) one value of k
(c) two values of k (d) none of these

55. If x, y, z are in A.P. , then the value of the det A, where $A=\begin{bmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{bmatrix}$ is
(a) 0 (b) 1
(c) 2 (d) none of these

56. If $\sin 2x + 1 = 0$, then $\begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix}^2$ equals
(a) 1 (b) 2
(c) 3 (d) none of these

57. Let λ and α be real. Let S denote the set of all values of λ for which the system of linear equations

$$\lambda x+(\sin\alpha)y+(\cos\alpha)z=0$$
$$x+(\cos\alpha)y+(\sin\alpha)z=0$$
$$-x+(\sin\alpha)y-(\cos\alpha)z=0$$

has a non-trivial solution, then S contains
(a) (−1, 1) (b) $[-\sqrt{2},\sqrt{2}]$
(c) $[1,\sqrt{2}]$ (d) (−2, 2)

58. If the system of equations $2x+ay+6z=8$, $x+2y+bz=5$ and $x+y+3z=4$ has a unique solution, then
(a) $a\neq 2, b\neq 3$ (b) $a\neq 3, b\neq 2$
(c) $a\neq 2, b=3$ (d) $a=2, b\neq 2$

59. The value of $\begin{bmatrix} x & x^2-yz & 1 \\ y & y^2-zx & 1 \\ z & z^2-xy & 1 \end{bmatrix}$ is
(a) 1 (b) −1
(c) 0 (d) $-xyz$

60. The repeated factor of the determinant $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$ is
(a) $x-y$ (b) $y-z$
(c) $z-x$ (d) none of these

61. The system of equations $x\sin\theta-\sqrt{3}y=0$ and $x\cos\theta + y = 0$ has an infinite number of solutions, for θ equal to
(a) $\pi/3$ (b) $2\pi/3$
(c) $5\pi/3$ (d) $4\pi/3$

62. If $p+q+r=0=a+b+c$, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

(a) 0 (b) $pa+qb+rc$
(c) 1 (d) none of these

63. If $\alpha = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$ and $= \beta = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$, then

(a) $\alpha \neq \beta$ (b) $\alpha = \beta$
(c) $\alpha = 2\beta$ (d) $\alpha = -\beta$

64. One factor of $\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & cb \\ ca & cb & c^2+x \end{vmatrix}$ is

(a) $1/x$
(b) $(a^2+x)(b^2+x)(c^2+x)$
(c) x^2
(d) $a^2+b^2+c^2+x$

EXERCISE SET 2

1. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = ?$

(a) abc (b) Σa
(c) 0 (d) $(\Sigma a)^2$

2. The determinant

$$\Delta = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{bmatrix}$$

is independent of

(a) α (b) β
(c) α and β (d) neither α nor β

3. The maximum and minimum values of (3×3) determinant whose elements belong to $\{0, 1\}$ are

(a) $1, -1$ (b) $2, -2$
(c) $4, -4$ (d) none of these

4. Let $f(x) = \begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$, then $f'\left(\frac{\pi}{2}\right) = ?$

(a) 8 (b) 6
(c) 4 (d) 2

5. The solution of equations $x+2y+3z=1$, $2x+y+3z=2$ and $5x+5y+9z=4$ is

(a) unique (b) infinity
(c) inconsistent (d) none of these

6. The value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ $(x, y, z$ being +ve) is

(a) $\log_y x$ (b) $\log_z y$
(c) $\log_x z$ (d) 0

7. If $p+q+r=0$, then $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

(a) 0 (b) pqr
(c) abc (d) $pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

8. The system of equations $ax+4y+z=0$, $bx+3y+z=0$, and $cx+2y+z=0$ has a non-trivial solution if a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

9. $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = ?$

(a) abc (b) $a^2b^2c^2$
(c) 0 (d) none of these

10. The value of the determinant $\Delta = \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$ is

(a) 2! (b) 3!
(c) 4! (d) 5!

11. The value of k for which the system of equations $3x+ky-2z=0$, $x+ky+3z=0$ and $2x+3y-4z=0$ has a non-trivial solution is

(a) 15 (b) 31/2
(c) 16 (d) 33/2

12. The value of the determinant (without expansion)

$$\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = ?$$

(a) abc (b) $a+b+c$
(c) 0 (d) $a^2+b^2+c^2$

13. If $a^2+b^2+c^2=0$ and $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = ka^2b^2c^2$, then the value of k is

(a) 1 (b) 2
(c) 3 (d) 4

14. The system of equations $2x - y + z = 0$, $x - 2y + z = 0$ and $\lambda x - y + 2z = 0$ has infinite number of non-trivial solutions for

(a) $\lambda = 1$ (b) $\lambda = 5$
(c) $\lambda = -5$ (d) no real value of λ

15. If a, b, c are respectively the pth, qth and rth terms of an H.P., then $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = ?$

(a) abc (b) $p+q+r$
(c) 0 (d) none of these

16. If $a \neq b \neq c$ such that $\begin{vmatrix} a^3-1 & b^3-1 & c^3-1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = 0$, then

(a) $ab+bc+ca=0$ (b) $a+b+c=0$
(c) $abc=1$ (d) $a+b+c=1$

17. The system of equations $2x + 3y = 8$, $7x - 5y + 3 = 0$ and $4x - 6y + \lambda = 0$ is solvable if λ is

(a) 6 (b) 8
(c) -8 (d) -6

18. If $U_n = \begin{vmatrix} n & 15 & 8 \\ n^2 & 35 & 9 \\ n^3 & 25 & 10 \end{vmatrix}$, then $\sum_{n=1}^{5} U_n = ?$

(a) 0 (b) 25
(c) 625 (d) none of these

19. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5+bx^4+cx^3+dx^2+\lambda x+\mu$ be an identity in x, where a, b, c, d, λ, μ are independent of x. Then the value of λ is

(a) 3 (b) 2
(c) 4 (d) none of these

20. If the solutions of the system of equations $3x - y + 4z - 3 = 0$, $x + 2y - 3z + 2 = 0$ and $6x + 5y + \lambda z + 3 = 0$ are infinite, then $\lambda = ?$

(a) 7 (b) -7
(c) 5 (d) -5

21. If $\Delta = \begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$, then Δ is equal to

(a) $(x+z-2y)^2$ (b) $(y+z-2x)^2$
(c) $(z+x-2y)^2$ (d) none of these

22. The value of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ is}$$

(a) $3\pi/24$ (b) $5\pi/24$
(c) $11\pi/24$ (d) $\pi/24$

23. The value of the determinant

$$\Delta = \begin{vmatrix} 2a_1b_1 & a_1b_2+a_2b_1 & a_1b_3+a_3b_1 \\ a_1b_2+a_2b_1 & 2a_2b_2 & a_2b_3+a_3b_2 \\ a_1b_3+a_3b_1 & a_3b_2+a_2b_3 & 2a_3b_3 \end{vmatrix} \text{ is}$$

(a) 1 (b) -1
(c) 0 (d) $a_1a_2a_3b_1b_2b_3$

24. If ω is an imaginary cube root of unity, then

$$\begin{vmatrix} \lambda+1 & \omega & \omega^2 \\ \omega & \lambda+\omega^2 & 1 \\ \omega^2 & 1 & \lambda+\omega \end{vmatrix} \text{ is equal to}$$

(a) 0 (b) $\lambda^3 + 1$
(c) λ^3 (d) none of these

25. If $\begin{vmatrix} x^n & x^{n+2} & x^{n+4} \\ y^n & y^{n+2} & y^{n+4} \\ z^n & z^{n+2} & z^{n+4} \end{vmatrix} = \left(\frac{1}{y^2}-\frac{1}{x^2}\right)\left(\frac{1}{z^2}-\frac{1}{y^2}\right)\left(\frac{1}{x^2}-\frac{1}{z^2}\right)$, then $n = ?$

(a) 4 (b) -4
(c) 2 (d) -2

26. If x, y, z are all distinct and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then the value of xyz is

(a) -2 (b) -1
(c) -3 (d) none of these

27. If $f(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$, then

$f(\theta) - 2f(\theta) + f(\theta) = ?$

(a) 1 (b) -1
(c) 0 (d) none of these

28. If $\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} = \lambda$, then $\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = ?$

(a) λ (b) 2λ
(c) $-\lambda$ (d) -2λ

29. The value of the determinant

$$\begin{vmatrix} \log_a\left(\frac{x}{y}\right) & \log_a\left(\frac{y}{z}\right) & \log_a\left(\frac{z}{x}\right) \\ \log_{a^2}\left(\frac{y}{z}\right) & \log_{a^2}\left(\frac{z}{x}\right) & \log_{a^2}\left(\frac{x}{y}\right) \\ \log_{a^3}\left(\frac{z}{x}\right) & \log_{a^3}\left(\frac{x}{y}\right) & \log_{a^3}\left(\frac{y}{z}\right) \end{vmatrix} \text{ is}$$

(a) 1 (b) -1
(c) 0 (d) 1/6 $\log_a xyz$

30. The value of $\begin{vmatrix} 1 & 1 & 1 \\ (2^x+2^{-x})^2 & (3^x+3^{-x})^2 & (5^x+5^{-x})^2 \\ (2^x-2^{-x})^2 & (3^x-3^{-x})^2 & (5^x-5^{-x})^2 \end{vmatrix}$ is

(a) 0 (b) 30^x
(c) 30^{-x} (d) none of these

31. The sum of the two non-integral roots of

$$\Delta = \begin{vmatrix} x & 3 & 4 \\ 5 & x & 5 \\ 4 & 2 & x \end{vmatrix} = 0 \text{ is}$$

(a) 4 (b) -4
(c) 16 (d) none of these

32. If $\theta \in (0, \pi/2)$, then the value of

$$\begin{vmatrix} (\sin\theta+\operatorname{cosec}\theta)^2 & (\sin\theta-\operatorname{cosec}\theta)^2 & 1 \\ (\cos\theta+\sec\theta)^2 & (\cos\theta-\sec\theta)^2 & 1 \\ (\tan\theta+\cot\theta)^2 & (\tan\theta-\cot\theta)^2 & 1 \end{vmatrix} = ?$$

(a) $\sin\theta + \cos\theta + \tan\theta$ (b) 1
(c) 0 (d) 4

33. If $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = \alpha a^2b^2c^2$, then the value of α is

(a) 4 (b) 3
(c) 2 (d) 1

34. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$

$$= \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = \lambda a^2b^2c^2, \text{ then } \lambda = ?$$

(a) 2 (b) 1
(c) 4 (d) 3

35. The only integral root of the equation

$$\begin{vmatrix} 2-y & 2 & 3 \\ 2 & 5-y & 6 \\ 3 & 4 & 10-y \end{vmatrix} = 0 \text{ is}$$

(a) $y = 2$ (b) $y = 3$
(c) $y = 1$ (d) none of these

36. If x, y, z are in A.P., then the value of the determinant

$$\begin{bmatrix} a+2 & a+3 & a+2x \\ a+3 & a+4 & a+2y \\ a+4 & a+5 & a+2z \end{bmatrix} \text{ is}$$

(a) $2a$ (b) a
(c) 1 (d) 0

37. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ?$

(a) $1 + \Sigma a$ (b) $1+\Sigma\frac{1}{a}$
(c) $abc\left[1+\Sigma\frac{1}{a}\right]$ (d) none of these

38. The determinant

$$\Delta = \begin{vmatrix} a^2 & a & 1 \\ \cos(nx) & \cos(n+1)x & \cos(n+2)x \\ \sin(nx) & \sin(n+1)x & \sin(n+2)x \end{vmatrix} \text{ is independent of}$$

(a) n (b) a
(c) x (d) none of these

39. If a, b and c are non-zero real numbers, then

$$\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} \text{ is equal to}$$

(a) $a^2b^2c^2$ (b) $bc + ca + ab$
(c) abc (d) 0

40. If $\begin{vmatrix} p & q-y & r-z \\ p-x & q & r-z \\ p-x & q-y & r \end{vmatrix} = 0$, then the value of $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}$ is

(a) 0 (b) 1
(c) 2 (d) 4 pqr

41. Let m be a positive integer and

$$\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix} \quad (0 \le r \le m)$$

Then the value of $\sum_{r=0}^{m} \Delta_r$ is given by

(a) 0 (b) $m^2 - 1$
(c) 2^m (d) $2^m \sin^2(2^m)$

42. If a, b and c are the roots of the equation $x^3 + px + q = 0$, ($p \neq 0$, $q \neq 0$) the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) $p^2 - 2q$ (b) p
(c) 0 (d) q

43. $\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} = ?$

(a) x^4 (b) 4
(c) 0 (d) −8

Assertion-Reason Type Questions

Each of these questions contains two statements: Statement 1 (Assertion) and Statement 2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b) (c) or (d) given below:

(a) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1
(b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1
(c) Statement 1 is true; Statement 2 is false
(d) Statement 1 is false; Statement 2 is true

44. Statement 1: The value of the determinant

$$A = \begin{vmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{vmatrix}$$ is equal to 0.

Statement 2: If A is a skew-symmetric determinant of odd order, then $|A|$ is a perfect square.

45. Statement 1: The value of $\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ is zero.

where $n \in N$.

Statement 2: $1 + \omega + \omega^2 = 0$

46. Statement 1: The minor of 5 in the determinant

$\begin{vmatrix} 2 & 5 & 8 \\ 0 & 3 & 7 \\ -1 & -2 & 6 \end{vmatrix}$ is −7.

Statement 2: The determinant that is left by cancelling the row and column intersecting at a particular element is called the minor.

47. Statement 1: If a, b, c are different and $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$, then $abc\,(ab + bc + ac) = a + b + c$.

Statement 2: $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (ab+bc+ca)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

48. Statement 1:

If $f(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ a & b & c \\ d & e & f \end{vmatrix}$, where f_1, f_2, f_3 are differentiable functions and a, b, c, d, e, f are constants, then

$$\int f(x)dx = \begin{vmatrix} \int f_1(x)dx & \int f_2(x)dx & \int f_3(x)dx \\ a & b & c \\ d & e & f \end{vmatrix} + k$$

Statement 2: Integration of the sum of several functions is equal to the sum of integrations of individual function.

49. Statement 1: If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is equal to $k^3\,\Delta$.

Statement 2: If any row of a determinant is multiplied by r, then the value of the determinant is also multiplied by r.

Previous Years' Questions

50. If $\omega (\neq 1)$ is a cubic root of unity, then

$$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix} \text{ equals}$$

(a) 0 (b) 1
(c) i (d) ω

51. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x+iy$, then

(a) $x=0, y=3$ (b) $x=0, y=0$
(c) $x=3, y=1$ (d) $x=1, y=3$

52. If 1, ω, ω^2 are the cube roots of unity,

then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to

(a) 0 (b) 1
(c) ω (d) ω^2

53. If the system of linear equations

$$x+2ay+az=0$$
$$x+3by+bz=0$$
$$x+4cy+cz=0$$

has a non-zero solution, then a, b, c

(a) are in A.P. (b) satisfy $a+2b+3c=0$
(c) are in H.P. (d) are in G.P.

54. If $a_1, a_2, a_3 \dots$ are in G.P., then

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} \text{ is equal to}$$

(a) 0 (b) 1
(c) 2 (d) 4

55. If $a^2+b^2+c^2=-2$ and

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix},$$

then $f(x)$ is a polynomial of degree

(a) 2 (b) 0
(c) 3 (d) 1

56. The system of equations $\alpha x+y+z=\alpha-1$, $x+\alpha y+z=\alpha-1$ and $x+y+\alpha z=\alpha-1$ has no solution, if α is

(a) either -2 or 1 (b) -2
(c) not -2 (d) 1

57. If $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then Δ is

(a) divisible by neither x nor y
(b) divisible by y but not by x
(c) divisible by x but not by y
(d) divisible by both x and y

58. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) -1 (b) 0
(c) 1 (d) 2

59. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true?

(a) If $\det(A) = \pm 1$, then $\underline{A}^{-1}$ need not exist
(b) If $\det(A) = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(c) If $\det(A) = \pm 1$, then A^{-1} exists and all its entries are non-integers
(d) If $\det(A) = \pm 1$, then A^{-1} exists and all its entries are integers

60. Let a, b, c be such that $(b+c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of n is

(a) zero (b) any even integer
(c) any odd integer (d) any integer

61. If x is a positive integer, then $\begin{vmatrix} x! & (x+1)! & (x+2)! \\ (x+1)! & (x+2)! & (x+3)! \\ (x+2)! & (x+3)! & (x+4)! \end{vmatrix}$

is equal to

(a) $2x!(x+1)!$ (b) $2x!(x+1)!(x+2)!$
(c) $2x!(x+3)!$ (d) $2(x+1)!(x+2)!(x+3)!$

62. Let $\omega = \dfrac{-1}{2} + i\dfrac{\sqrt{3}}{2}$, then the value of the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} \text{ is}$$

(a) 3ω (b) $3\omega(\omega-1)$
(c) $3\omega^2$ (d) $3\omega(1-\omega)$

63. If $C = 2\cos\theta$, then the value of the determinant

$$\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 6 & 1 & C \end{vmatrix} \text{ is}$$

(a) $\frac{\sin 4\theta}{\sin\theta}$ (b) $4\cos^2\theta(\cos\theta - 1)$
(c) $\frac{2\sin^2 2\theta}{\sin\theta}$ (d) none of these

64. If $D_r = \begin{vmatrix} 2^{r-1} & 2\cdot 3^{r-1} & 4\cdot 5^{r-1} \\ \alpha & \beta & \gamma \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$, then the vlaue of $\sum_{r=1}^{n} D_r$ is equal to

(a) $\alpha + \beta + \gamma$ (b) $\alpha \cdot 2^n + \beta \cdot 3^n + \gamma \cdot 4^n$
(c) 0 (d) $\alpha\beta\gamma$

65. If $\begin{vmatrix} -12 & 0 & \lambda \\ 0 & 2 & -1 \\ 2 & 1 & 15 \end{vmatrix} = -360$, then the value of λ is

(a) –1 (b) –2 (c) –3 (d) 4

66. If $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ \sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$, then Δ lies in the interval

(a) [2, 4] (b) [3, 4] (c) [1, 4] (d) none of these

67. The value of the determinant $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ is equal to

(a) $6xyz$ (b) xyz
(c) $4xyz$ (d) $xy + yz + zx$

68. **Statement 1:** The determinant of a skew-symmetric matrix of order 3 is zero.
Statement 2: For any matrix A, $\det(A)^T = \det(A)$ and $\det(-A) = -\det(A)$, where det(2) denotes the determinant of matrix B. Then
(a) both statements are true
(b) both statements are false
(c) Statement 1 is false and Statement 2 is true
(d) Statement 1 is true and Statement 2 is false

ANSWERS

Exercise Set 1

1. (a)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (b)
11. (a)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (c)
21. (d)	**22.** (b)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (d)
31. (c)	**32.** (b)	**33.** (a)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (c)	**38.** (c)	**39.** (d)	**40.** (a)
41. (c)	**42.** (b)	**43.** (c)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (c)	**49.** (a)	**50.** (c)
51. (a)	**52.** (c)	**53.** (c)	**54.** (b)	**55.** (a)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (c)	**60.** (c)
61. (d)	**62.** (a)	**63.** (b)	**64.** (c) and (d)						

Exercise Set 2

1. (c)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (c)
11. (d)	**12.** (c)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (d)
21. (a)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (c)	**28.** (c)	**29.** (c)	**30.** (a)
31. (b)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (c)	**36.** (d)	**37.** (c)	**38.** (a)	**39.** (d)	**40.** (c)
41. (a)	**42.** (c)	**43.** (d)	**44.** (c)	**45.** (a)	**46.** (d)	**47.** (a)	**48.** (a)	**49.** (a)	**50.** (a)
51. (b)	**52.** (a)	**53.** (c)	**54.** (a)	**55.** (a)	**56.** (b)	**57.** (d)	**58.** (c)	**59.** (d)	**60.** (c)
61. (b)	**62.** (b)	**63.** (d)	**64.** (c)	**65.** (c)	**66.** (a)	**67.** (c)	**68.** (d)		

To be continued ...

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