



**ST. LAWRENCE HIGH SCHOOL**  
A JESUIT CHRISTIAN MINORITY INSTITUTION



**STUDY MATERIAL-17**

**SUBJECT – STATISTICS**

**Pre-test**

Chapter: TIME SERIES ANALYSIS

Class: XII

Topic: DETERMINATION OF TREND

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# TIME SERIES ANALYSIS

## PART 2

**Measurement of trend:**

Trend can be measured by eliminating the other component. The methods which are generally used are i) free hand curve fitting ii) semi average method iii) moving average method iv) least square method or mathematical curve fitting method.

**Moving average method:**

In this technique, a series of arithmetic means, each of  $m$  successive observations of given data, is computed and these means are referred to as the moving average of period  $m$ , where  $m$  is average period of the cycles or multiple of it. To begin with, we take first  $m$  values; at the next stage, exclude the first and include the  $(m+1)$ th value and so on. We repeat this process until we reach at the last set of  $m$  values. Each mean is placed against the mid point of the time interval it covers. If  $m$  is odd, the moving averages corresponded to the tabulated time for which time series is given. On the other hand when  $m$  is even, each moving average falls midway between two tabulated time values. In this case a subsequent 2- item moving average is computed to make the resulting moving average values correspond to given times. These moving averages are trend values for corresponding times.

**Merits:**

i) The method is flexible. Any change in the trend is faithfully reflected by the moving averages.

ii) It does not involve complicated calculations. Further, the inclusion of few additional observations to the given series simply results in the computation of some more averages, previous calculations remain unaffected.

iii) The method of moving averages causes the merit of objectivity since the period of moving average can be more or less objectively determined.

#### **Demerits:**

i) In this method some trend values at each end of the series cannot be determined as we do not get the trend values for first  $m$  years and last  $m$  years in case of  $2m-1$  and also  $2m$ .

ii) Because of the previous defects moving average cannot be used in forecasting.

#### **Method of mathematical curve fitting:**

It is the most important and rational method of determining trend. Here we first choose a suitable trend equation from the graphical representation of data and then estimate the constants associated with the equation on the basis of available data.

##### **Case 1 – Linear trend**

We define the trend equation as  $y_t = a + bt$

The constance a & b are estimated by method of least squares. Suppose we are given  $y_t$  for n equidistant values of t. the method of least square requires that a & b should be determined that

Normal equations:

$$\sum_{t=1}^n y_t = na + b \sum_{t=1}^n t$$

$$\sum_{t=1}^n ty_t = a \sum_{t=1}^n t + b \sum_{t=1}^n t^2$$

## Case - 2 - exponential form

Next proceeding as before, A & B are estimated. The anti logarithms of these estimates give those of a & b and hence, the exponential trend equation is fitted to the time series.

Equation taken  $y_t = ab^t$

Taking logarithm with base e both sides,

$$\ln y_t = \ln a + t \cdot \ln b$$

$$Y_t = A + Bt$$

Where,

$$Y_t = \ln y_t, A = \ln a, B = \ln b$$

So the normal equations are

$$\sum_{t=1}^n Y_t = nA + B \sum_{t=1}^n t$$
$$\sum_{t=1}^n tY_t = A \sum_{t=1}^n t + B \sum_{t=1}^n t^2$$

Case 3 –Parabolic trend

We define the trend equation as  $y_t = a + bt + ct^2$

The constance a, b & c are estimated by method of least squares.

Suppose we are given  $y_t$  for n equidistant values of t. the method of least square requires that a & b should be determined that

Normal equations:

$$\sum_{t=1}^n y_t = na + b \sum_{t=1}^n t + c \sum_{t=1}^n t^2$$
$$\sum_{t=1}^n ty_t = a \sum_{t=1}^n t + b \sum_{t=1}^n t^2 + c \sum_{t=1}^n t^3$$
$$\sum_{t=1}^n t^2 y_t = a \sum_{t=1}^n t^2 + b \sum_{t=1}^n t^3 + c \sum_{t=1}^n t^4$$

**Remarks:**

The successive points of time be generally be equidistant, hence considering the mid point of the entire time span as origin, sum of  $t$  values can be reduced to zero. If  $d$  be the common difference between two successive time values, then, depending on odd or even number of values, one may take  $d$  or  $d/2$  as the new unit for  $t$  for case of computations.

**Merits:**

- i) this method is most objective, since the appropriate curve to be fitted can be objectively determined through graphical representation of data or otherwise.
- ii) this method enables one to obtain the trend values for all the given points of time.
- iii) the method can be used for predicting future trend as it assumes the a of change.

**Demerits:**

- i) The addition of a single observation to the given data necessitates all calculation to be done afresh.
- ii) It involves time consuming calculation unless the trend equation is simple linear or quadratic.
- iii) the method is rigid. If there are sharp changes in the trend then to use the method, separate trend equation are to be fitted to different parts.

**Reduction of trend equations:**

In case of reduction of trend equation to a smaller duration as time period, all the values should be divided by the number of time periods which can be fitted in the old scale and to convert it to a time period which is greater than the previous then all the values should be multiplied by the number of time periods fitted in the new scale. In case of the new scale the origin should be determined as the mid point of the duration of any time period. For that some constant in terms of the new scale to be added or subtracted with the  $t$  to get the exact trend value.

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