ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-12

SUBJECT – STATISTICS

1st term

Chapter: DISPERSION **Class: XI**

Topic: Standard Deviation **Date: 17.08.2020**

DISPERSION

PART 3

Definition : Mean square deviation about an arbitrary central value 'A' is defined as the arithmetic mean of the square of the differences of central value 'A' from all the observations.

For ungrouped grouped data

Observations: $x_1 , x_2 , \dots \dots \dots \dots . , x_n$

$$MSD_A(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^2$$

For grouped data

Observations: $x_1 , x_2 , \dots \dots \dots \dots . , x_n$

Frequency: $f_1 , f_2 , \dots \dots . \dots \dots . , f_n$

$$MSD_A(x) = \frac{1}{N}\sum_{i=1}^{n}(x_i - A)^2 f_i$$

Properties:

- Mean square deviation is least when taken about mean.

$$MSD_A(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^2$$

$$\frac{d}{dA}MSD_A(x) = \frac{1}{n}\sum_{i=1}^{n} 2\,(x_i - A)(-1)$$

$$\frac{d^2}{dx^2}MSD_A(x) = 2 > 0$$

So $MSD_A(x)$ $is\ minimum$ when $A = \bar{x}$

This minimum mean square deviation about mean is known as variance of x.

Variance of x is then defined as

For ungrouped grouped data

Observations: $x_1, x_2, \ldots \ldots \ldots \ldots., x_n$

$$s_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

For grouped data

Observations: $x_1, x_2, \ldots \ldots \ldots \ldots., x_n$

Frequency: $f_1, f_2, \ldots \ldots. \ldots \ldots., f_n$

$$s_x^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \bar{x})^2 f_i$$

To get back the same unit the positive square root of the variance is taken and is known as the standard deviation.

For ungrouped grouped data

Observations: $x_1, x_2, \ldots \ldots \ldots \ldots., x_n$

$$s_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

For grouped data

Observations: x_1 , x_2 , … … … … . , x_n

Frequency: f_1 , f_2 , … … . … … . , f_n

$$s_x = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(x_i - \bar{x})^2 f_i}$$

- If $y_i = a + b\, x_i$, $\forall\, i = 1(1)n$

Then, $s_y^2 = b^2 s_x^2$

And $s_y = |b| s_x$

Proof: By definition

$$s_y^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}(a + bx_i - a - b\overline{x)}^2$$

$$= b^2 \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= b^2 s_x^2$$

In case of grouped data

$$s_y^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 f_i$$

$$= \frac{1}{n}\sum_{i=1}^{n}(a + bx_i - a - b\overline{x)}^2 f_i$$

$$= b^2 \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 f_i$$

$$= b^2 s_x^2$$

- If $x_i = k(constant)$

Then $s_x^2 = 0$

Proof:

$$s_x^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$\Rightarrow s_x^2 = \frac{1}{n}\sum_{i=1}^{n}(k - k)^2$$

= 0

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