ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-13

SUBJECT – STATISTICS

Pre-test

Chapter: THEORITICAL PROBABILITY DISTRIBUTION **Class: XII**

Topic: POISSON PROBABILITY DISTRIBUTION **Date: 25.06.20**

PROBABILITY DISTRIBUTION

PART 7

A random variable X follows Poisson distribution with parameter λ

$$X \sim \text{poissn}(\lambda)$$

The pmf of the random variable X is given by

$$f(x) = \frac{e^{-\lambda}\,\lambda^{x}}{x!}\ , x = 0(1)\infty$$

Condition:

A random variable X follows Poisson distribution when

i. Number of trials are countably infinite

ii. In every trial there exist only two possible outcomes , viz, success and failure.

iii. Probability of success remains same in each single trial.

PROPERTIES:

1. **So we can say that basically Poisson distribution is a limiting case of Binomial probability distribution.**

Proof:

Consider n: number of trials

p: probability of success in each trial

In this proof we use the assumption that n is too large, ie, n→ ∞, and p is too small, ie, p→ 0, such that np is a constant = λ (say).

$$\lim_{n\to\infty} n_{c_x} p^x (1-p)^{n-x}$$

$$= \lim_{n\to\infty} \frac{n\,(n-1)(n-2)..............(n-x+1)}{x!}\; p^x (1-p)^{n-x}$$

$$= \frac{1}{x!} \lim_{n\to\infty} 1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)...\left(1-\frac{x-1}{n}\right) n^x p^x (1-\frac{np}{n})^{n-x}$$

Putting np = λ,

$$= \frac{\lambda^x}{x!} \frac{\lim_{n\to\infty}(1-\frac{\lambda}{n})^n}{\lim_{n\to\infty}(1-\frac{\lambda}{n})^x}$$

$$= \frac{\lambda^x}{x!}\, e^{-\lambda}$$

Since $\lim_{n\to\infty} 1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)...\left(1-\frac{x-1}{n}\right) = 1$

$$\lim_{n\to\infty}(1-\frac{\lambda}{n})^x = 1$$

$$\lim_{n\to\infty}(1-\frac{\lambda}{n})^n = e^{-\lambda}$$

So Poisson distribution is a limiting case of Binomial probability distribution.

2. **Expectation of X, E(X)=** λ

$$E(X) = \sum_{x=0}^{\infty} x. f(x)$$

$$= \sum_{x=0}^{\infty} x.\ e^{-\lambda.} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda.}\, \lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= e^{-\lambda.}\, \lambda\, e^{\lambda.} = \lambda$$

3. **Variance of X, V(X)=** λ

$$E(X(X-1)) = \sum_{x=0}^{\infty} x(x-1). f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1).\ e^{-\lambda.} \frac{\lambda^x}{x!}$$

$$= e^{-\lambda.}\, \lambda^2 \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}$$

$$= e^{-\lambda.}\ \lambda^2\, e^{\lambda.} = \lambda^2$$

Now V(X) = $E(X(X-1)) + E(X) - (E(X))^2$

$= \lambda^2 + \lambda - \lambda^2 = \lambda$

So in Poisson E(X) = V(X)= λ (parameter of the distribution)

Eg, A police man claims that the average number of accidents during a month at a junction is two on an average. Find the probability that during a particular month four accidents would take place.

Explanation:

In this problem occurrence of accident is the success from the problem we cannot say that what may be the maximum possible number of success. It is basically countably infinite. So we use Poisson distribution.

If we compare the problem with another one to find the probability of number of heads in five tosses of a fair coin where getting head is success. In this problem we can easily say the maximum possible number of success is five. Hence it is a problem of Binomial distribution.

Solution:

Define X: Number of accidents during a month

$X \sim \text{poissn}(2)$

So $f(x) = \frac{e^{-2}\, 2^x}{x!}$ $, x = 0(1)\infty$

Required probability $f(4) = \frac{e^{-2}\, 2^4}{4!} = \frac{2}{3e^2}$

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