STUDY MATERIAL-10

SUBJECT – MATHEMATICS

Pre-test

Chapter: Limit **Class: XII**

Topic: Limit **Date: 12.06.2020**

-: LIMIT :-

➢ Some Important Solved Problems :-

1. Evaluate the limit $\lim\limits_{t\to 4}\dfrac{t-\sqrt{3t+4}}{4-t}$

$$\lim_{t\to 4}\frac{t-\sqrt{3t+4}}{4-t}=\lim_{t\to 4}\frac{\left(t-\sqrt{3t+4}\right)}{(4-t)}\,\frac{\left(t+\sqrt{3t+4}\right)}{\left(t+\sqrt{3t+4}\right)}$$

$$=\lim_{t\to 4}\frac{t^2-(3t+4)}{(4-t)\left(t+\sqrt{3t+4}\right)}$$

$$=\lim_{t\to 4}\frac{t^2-3t-4}{(4-t)\left(t+\sqrt{3t+4}\right)}$$

$$=\lim_{t\to 4}\frac{(t-4)(t+1)}{-(t-4)\left(t+\sqrt{3t+4}\right)}$$

$$=\lim_{t\to 4}\frac{t+1}{-\left(t+\sqrt{3t+4}\right)}$$

$$=-\frac{5}{8}$$

2. Evaluate $\lim_{t \to -3} \frac{6+4t}{t^2+1}$

$$\lim_{t \to -3} \frac{6+4t}{t^2+1} = \frac{-6}{10} = \boxed{-\frac{3}{5}}$$

3. Evaluate $\lim_{x \to -5} \frac{x^2-25}{x^2+2x-15}$

$$\lim_{x \to -5} \frac{x^2-25}{x^2+2x-15} = \lim_{x \to -5} \frac{(x-5)(x+5)}{(x-3)(x+5)} = \lim_{x \to -5} \frac{x-5}{x-3} = \boxed{\frac{5}{4}}$$

4. Evaluate $\lim_{z \to 8} \frac{2z^2-17z+8}{8-z}$

$$\lim_{z \to 8} \frac{2z^2-17z+8}{8-z} = \lim_{z \to 8} \frac{(2z-1)(z-8)}{-(z-8)} = \lim_{z \to 8} \frac{2z-1}{-1} = \boxed{-15}$$

5. Evaluate $\lim_{y \to 7} \frac{y^2-4y-21}{3y^2-17y-28}$

$$\lim_{y \to 7} \frac{y^2-4y-21}{3y^2-17y-28} = \lim_{y \to 7} \frac{(y-7)(y+3)}{(3y+4)(y-7)} = \lim_{y \to 7} \frac{y+3}{3y+4} = \frac{10}{25} = \boxed{\frac{2}{5}}$$

6. Evaluate $\lim_{h \to 0} \frac{(6+h)^2-36}{h}$

$$\lim_{h \to 0} \frac{(6+h)^2-36}{h} = \lim_{h \to 0} \frac{36+12h+h^2-36}{h}$$

$$= \lim_{h \to 0} \frac{h(12+h)}{h} = \lim_{h \to 0}(12+h) = \boxed{12}$$

7. Evaluate $\lim_{z\to 4}\dfrac{\sqrt{z}-2}{z-4}$

$$\lim_{z\to 4}\frac{\sqrt{z}-2}{z-4}=\lim_{z\to 4}\frac{\left(\sqrt{z}-2\right)}{(z-4)}\frac{\left(\sqrt{z}+2\right)}{\left(\sqrt{z}+2\right)}$$

$$=\lim_{z\to 4}\frac{z-4}{(z-4)\left(\sqrt{z}+2\right)}=\lim_{z\to 4}\frac{1}{\sqrt{z}+2}=\boxed{\frac{1}{4}}$$

8. Evaluate $\lim_{x\to -3}\dfrac{\sqrt{2x+22}-4}{x+3}$

$$\lim_{x\to -3}\frac{\sqrt{2x+22}-4}{x+3}=\lim_{x\to -3}\frac{\left(\sqrt{2x+22}-4\right)}{(x+3)}\frac{\left(\sqrt{2x+22}+4\right)}{\left(\sqrt{2x+22}+4\right)}$$

$$=\lim_{x\to -3}\frac{2x+22-16}{(x+3)\left(\sqrt{2x+22}+4\right)}$$

$$=\lim_{x\to -3}\frac{2(x+3)}{(x+3)\left(\sqrt{2x+22}+4\right)}=\lim_{x\to -3}\frac{2}{\sqrt{2x+22}+4}=\frac{2}{8}=\boxed{\frac{1}{4}}$$

9. Evaluate $\lim_{x\to 0}\dfrac{x}{3-\sqrt{x+9}}$

$$\lim_{x\to 0}\frac{x}{3-\sqrt{x+9}}=\lim_{x\to 0}\frac{x}{\left(3-\sqrt{x+9}\right)}\frac{\left(3+\sqrt{x+9}\right)}{\left(3+\sqrt{x+9}\right)}=\lim_{x\to 0}\frac{x\left(3+\sqrt{x+9}\right)}{9-(x+9)}$$

$$=\lim_{x\to 0}\frac{x\left(3+\sqrt{x+9}\right)}{-x}=\lim_{x\to 0}\frac{3+\sqrt{x+9}}{-1}=\boxed{-6}$$

10. Given the function

$$f(x) = \begin{cases} 7 - 4x & x < 1 \\ x^2 + 2 & x \geq 1 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim\limits_{x \to -6} f(x)$ **(b)** $\lim\limits_{x \to 1} f(x)$

(a) $\lim\limits_{x \to -6} f(x)$

$$\lim_{x \to -6} f(x) = \lim_{x \to -6} (7 - 4x) = \boxed{31}$$

(b) $\lim\limits_{x \to 1} f(x)$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (7 - 4x) = \underline{3} \qquad \text{because } x \to 1^- \text{ implies that } x < 1$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \left(x^2 + 2\right) = \underline{3} \qquad \text{because } x \to 1^+ \text{ implies that } x > 1$$

So, in this case, we can see that, $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = 3$

and so we know that the overall limit must exist and, $\lim\limits_{x \to 1} f(x) = \boxed{3}$

11. Evaluate $\lim\limits_{x \to 5} (10 + |x - 5|)$

we can see that,

$$|x - 5| = \begin{cases} x - 5 & x \geq 5 \\ -(x - 5) & x < 5 \end{cases}$$

$$\lim_{x \to 5^-} (10 + |x - 5|) = \lim_{x \to 5^-} (10 - (x - 5)) = \lim_{x \to 5^-} (15 - x) = 10 \qquad \text{recall } x \to 5^- \text{ implies } x < 5$$

$$\lim_{x \to 5^+} (10 + |x - 5|) = \lim_{x \to 5^+} (10 + (x - 5)) = \lim_{x \to 5^+} (5 + x) = 10 \qquad \text{recall } x \to 5^+ \text{ implies } x > 5$$

So, for this problem, we can see that, $\lim\limits_{x\to 5^-}(10+|x-5|)=\lim\limits_{x\to 5+}(10+|x-5|)=10$

and so the overall limit must exist and, $\lim\limits_{x\to 5}(10+|x-5|)=\boxed{10}$

➢ HOMEWORK :-

1. $\lim\limits_{x\to -9}\left(1-4x^3\right)$

2. $\lim\limits_{y\to 1}\left(6y^4-7y^3+12y+25\right)$

3. $\lim\limits_{t\to 0}\frac{t^2+6}{t^2-3}$

4. $\lim\limits_{z\to 4}\frac{6z}{2+3z^2}$

5. $\lim\limits_{w\to -2}\frac{w+2}{w^2-6w-16}$

6. $\lim\limits_{t\to -5}\frac{t^2+6t+5}{t^2+2t-15}$

7. $\lim\limits_{x\to 3}\frac{5x^2-16x+3}{9-x^2}$

8. $\lim\limits_{z\to 1}\frac{10-9z-z^2}{3z^2+4z-7}$

9. $\lim\limits_{x\to -2}\frac{x^3+8}{x^2+8x+12}$

10. $\lim\limits_{t\to 8}\frac{t\,(t-5)-24}{t^2-8t}$

11. $\lim_{w\to -4} \dfrac{w^2-16}{(w-2)(w+3)-6}$

12. $\lim_{h\to 0} \dfrac{(2+h)^3-8}{h}$

13. $\lim_{h\to 0} \dfrac{(1+h)^4-1}{h}$

14. $\lim_{t\to 25} \dfrac{5-\sqrt{t}}{t-25}$

15. $\lim_{x\to 2} \dfrac{x-2}{\sqrt{2}-\sqrt{x}}$

16. $\lim_{z\to 6} \dfrac{z-6}{\sqrt{3z-2}-4}$

17. $\lim_{z\to -2} \dfrac{3-\sqrt{1-4z}}{2z+4}$

18. $\lim_{t\to 3} \dfrac{3-t}{\sqrt{t+1}-\sqrt{5t-11}}$

19. $\lim_{x\to 7} \dfrac{\frac{1}{7}-\frac{1}{x}}{x-7}$

20. $\lim_{y\to -1} \dfrac{\frac{1}{4+3y}+\frac{1}{y}}{y+1}$

21 Given the function

$$g(x) = \begin{cases} 5x + 24 & x < -3 \\ x^2 & -3 \le x < 4 \\ 1 - 2x & x \ge 4 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{x \to -3} g(x)$ **(b)** $\lim_{x \to 0} g(x)$ **(c)** $\lim_{x \to 4} g(x)$ **(d)** $\lim_{x \to 12} g(x)$

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