ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-19

SUBJECT – MATHEMATICS

1st - Term

Chapter: Coordinate Geometry **Class: XI**

Topic: Straight Lines **Date: 17.08.2020**

Question

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the points (2, 3).

Solution

The equation of a line in the intercept form is

$\frac{x}{a}+\frac{y}{b}=1$ ....(i)

Here, a and b are the intercepts on x and y axes respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that a = b.

Accordingly, equation (i) reduces to

$\frac{x}{a}+\frac{y}{a}=1$

$\Rightarrow x+y=a \quad ...(ii)$

Since the given line passes through point (2, 3), equation (ii) reduces to 2 + 3 = a $\Rightarrow$ a = 5

On substituting the value of a in equation (ii), we obtain

x + y = 5, which is the required equation of the line.

Question

Find the equation of the line passing through the points (2, 2) and cutting off intercepts on the axes whose sum is 9.

Solution

The equation of a line in the intercept form is

$\frac{x}{a}+\frac{y}{b}=1$ ...(i)

Here, a and b are the intercepts on x and y axes respectively.

It is given that a + b = 9 $\Rightarrow$ b = 9 – a ...(ii)

From equation (i) and (ii), we obtain

$\frac{x}{a}+\frac{y}{9-a}=1$ ...(iii)

It is given that the line passes through point (2, 2). Therefore, equation (iii) reduces to

$$\frac{2}{a}+\frac{2}{9-a}=1$$
$$\Rightarrow 2\left(\frac{1}{a}+\frac{1}{9-a}\right)=1$$
$$\Rightarrow 2\left(\frac{9-a+a}{a(9-a)}\right)=1$$
$$\Rightarrow \frac{18}{9a-a^2}=1$$
$$\Rightarrow 18=9a-a^2$$
$$\Rightarrow a^2-9a+18=0$$
$$\Rightarrow a^2-6a-3a+18=0$$
$$\Rightarrow a(a-6)-3(a-6)=0$$
$$\Rightarrow (a-6)(a-3)=0$$
$$\Rightarrow a=6 \text{ or } a=3$$

If a = 6 and b = 9 – 6 = 3, then the equation of the line is

$$\frac{x}{6}+\frac{y}{3}=1 \Rightarrow x+2y-6=0$$

If a = 3 and b = 9 – 3 = 6, then the equation of the line is

$$\frac{x}{3}+\frac{y}{6}=1 \Rightarrow 2x+y-6=0$$

Question

Find equation of the line through the points (0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of the line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution

The slope of the line making an angle $\frac{2\pi}{3}$ with the positive x-axis is $m=\tan\left(\frac{2\pi}{3}\right)=-\sqrt{3}$

Now, the equation of the line passing through points (0, 2) and having a slope $-\sqrt{3}$ is

$$(y-2)=-\sqrt{3}(x-0)$$
$$i.e., \sqrt{3}x+y-2=0$$

The slope of line parallel to line $\sqrt{3}x+y-2=0$ is $-\sqrt{3}$.

It is given that the line parallel to line $\sqrt{3}x+y-2=0$ crosses the y-axis 2 units below the origin i.e., it passes through point (0, 2).

Hence, the equation of the line passing through points (0, 2) and having a slope $-\sqrt{3}$ is

$$y-(-2)=-\sqrt{3}(x-0)$$
$$y+2=-\sqrt{3}x$$
$$\sqrt{3}x+y+2=0$$

Question

The perpendicular from the origin to a line meets it at the point (-2, 9), find the equation of the line.

Solution

The slope of the line joining the origin (0, 0) and point (-2, 9) is $m_1 = \frac{9-0}{-2-0} = -\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and points (-2, 9) is

$$m_2 = \frac{1}{m_1} = -\frac{1}{\left(-\frac{9}{2}\right)} = \frac{2}{9}$$

Now, the equation of the line passing through point (-2, 9) and having a slope m_2 is

$$(y-9) = \frac{2}{9}(x+2)$$

$$9y - 81 = 2x + 4$$

i.e., 2x-9y+85=0

Question

The perpendicular from the origin to a line meets it at the point (-2, 9), find the equation of the line.

Solution

The slope of the line joining the origin (0, 0) and point (-2, 9) is $m_1 = \frac{9-0}{-2-0} = -\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and points (-2, 9) is

$$m_2 = \frac{1}{m_1} = -\frac{1}{\left(-\frac{9}{2}\right)} = \frac{2}{9}$$

Now, the equation of the line passing through point (-2, 9) and having a slope m_2 is

$$(y-9) = \frac{2}{9}(x+2)$$

$$9y - 81 = 2x + 4$$

i.e., 2x-9y+85=0

Question

Find the equation of the line perpendicular to the line x - 7y + 5 = 0 and having *x* intercept 3.

Solution

The given equation of the line is x - 7y + 5 = 0.

Or, $y = \frac{1}{7}x + \frac{5}{7}$, which is of the form y = mx + c

$\therefore$ Slope of the given line = $\frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is $m = -\frac{1}{\left(\frac{1}{7}\right)} = -7$

The equation of the line with slope -7 and x-intercept 3 is given by
y = m (x – d)
$\Rightarrow y = -7(x-3)$
$\Rightarrow y = -7x+21$
$\Rightarrow 7x+y = 21$

Question

Find equation of the line parallel to the line 3x – 4y + 2 = 0 and passing through the point (-2, 3).

Solution

The equation of the given line is
3x – 4y + 2 = 0
Or y = $\frac{3x}{4}+\frac{2}{4}$
or y=$\frac{3}{4}x+\frac{1}{2}$, Which is of the form y = mx + c
$\therefore$ Slope of the given line = $\frac{3}{4}$
It is known that parallel lines have the same slope.
$\therefore$ Slope of the other line = m = $\frac{3}{4}$
Now, the equation of the line that has a slop of $\frac{3}{4}$ and passes through the points (-2, 3) is

$$(y-3)=\frac{3}{4}\{x-(-2)\}$$
$$4y-12=3x+6$$
i.e., 3x-4y+18=0

Question

Find angles between the lines $\sqrt{3}x+y=1$ and x+$\sqrt{3}y=1$.

Solution

The given lines are $\sqrt{3}x+y=1$ and x+$\sqrt{3}y=1$

$y=-\sqrt{3}x+1$ ...(1) and y=-$\frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}}$ ...(2)

The slope of line (1) is $m_1=-\sqrt{3}$, while the slope of line (2) is $m_2=-\frac{1}{\sqrt{3}}$.

The acute angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right|$$

$$\tan\theta = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2\times\sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^o$$

Thus, the angle between the given lines is either 30^o or $180^o - 30^o = 150^o$.

Question

Find the distance of the points (-1, 1) from the line 12(x + 6) = 5(y – 2).

Solution

The given equation of the line is 12(x + 6) = 5(y – 2).

$\Rightarrow$ 12x + 72 = 5y – 10

$\Rightarrow$ 12x – 5y + 82 = 0 ...(1)

On comparing equation (1) with general equation of line Ax + By + C = 0, we obtain A = 12, B = - 5, and C = 82.

It is known that the perpendicular distance (d) of a line Ax + By + C = 0 from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

The given point is (x_1, y_1) = (-1, 1).

Therefore, the distance of point (-1, 1) from the given line

$$= \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} \text{ units} = \frac{|-12 - 5 + 82|}{\sqrt{169}} \text{ units} = \frac{|65|}{13} \text{ units} = 5 \text{ units}$$

Prepared by :-

Mr. SUKUMAR MANDAL (SkM)