ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-15

SUBJECT – STATISTICS

Pre-test

Chapter: THEORITICAL PROBABILITY DISTRIBUTION **Class: XII**

Topic: UNIFORM PROBABILITY DISTRIBUTION **Date: 30.06.20**

PROBABILITY DISTRIBUTION

PART 9

This distribution occurs when the different values of random variables are equally probable. Suppose for instance that an unbiased die is rolled and the random variable X denotes the number of points on the face of the die. Then X has a uniform distribution, because it takes values 1, 2, …, 6, each with probability $\frac{1}{6}$. In general, this distribution is defined by the probability mass function

$$f(x) = \frac{1}{n}, \forall\, x = a(h)(n-1)h$$

Where a and h are fixed real numbers and n is a fixed positive integer.

Obviously, $f(x) \geq 0, \forall\, x$

And

$$\sum_{x=a}^{(n-1)h} f(x) = \sum_{x=a}^{(n-1)h} \frac{1}{n} = 1$$

PROPERTIES:

Note that

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

$$\sum_{i=0}^{n-1} i^2 = \frac{n(n-1)(2n-1)}{6}$$

Mean = $\mu = E(X)$

$$= \frac{1}{n} \sum_{i=0}^{n-1} (a + ih)$$

$$= \frac{1}{n} \{ na + h \sum_{i=0}^{n-1} i\}$$

$$= a + \frac{1}{2} h(n-1)$$

2. Variance of X = $\sigma^2 = E(X-\mu)^2$

$$= \sum_{x=0}^{n-1} (x-\mu)^2 f(x)$$

$$= \frac{1}{n} \sum_{i=0}^{n-1} [(a+ih) - \left\{a + \frac{1}{2} h(n-1)\right\}]^2$$

$$= \frac{h^2}{n} \sum_{i=0}^{n-1} [(a+ih) - \left\{a + \frac{1}{2} h(n-1)\right\}]^2$$

$$= \frac{h^2}{n} \{\sum_{i=0}^{n-1} \{i - \frac{1}{2}(n-1)\}^2$$

$$= \frac{h^2}{n} \{\sum_{i=0}^{n-1} i^2 - (n-1) \sum_{i=0}^{n-1} i + \frac{n(n-1)^2}{4}\}$$

$$= \frac{h^2}{n}\{\frac{1}{6}\ n(n-1)(2n-1) - \frac{1}{2}\ n(n-1)^2 + \frac{1}{4}\ n(n-1)^2\}$$

$$= \frac{h^2}{n}\{\frac{1}{6}\ (n-1)(2n-1) - \frac{1}{4}\ (n-1)^2\}$$

$$= \frac{h^2}{12}(n-1)\{2(2n-1) - 3(n-1)\}$$

$$= \frac{h^2}{12}(\,n^2 - 1)$$

3. $\mu_3 = E(X-\mu)^3$

$$= \frac{h^3}{n}\ \{\sum_{i=0}^{n-1}\{i - \frac{1}{2}(n-1)\}^3$$

$$= \frac{h^3}{n}\ \{\sum_{i=0}^{n-1} i^3 - \frac{3}{2}(n-1)\sum_{i=0}^{n-1} i^2 + \frac{3}{4}(n-1)^2\sum_{i=0}^{n-1} i - \frac{1}{8}n(n-1)^3\}$$

Putting the values of different sums and simplifying, we get

$\mu_3 = 0$

Similarly,

$\mu_4 = E(X-\mu)^4$

$$= \frac{h^4}{n}\ \{\sum_{i=0}^{n-1}\left\{i - \frac{1}{2}(n-1)\right\}^4$$

$$= \frac{h^4}{240}\ (n^2-1)(3n^2-7)$$

So $\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{144}{240}\ \frac{(3n^2-7)}{(n^2-1)}$$

$$= 1.8\ \frac{n^2 - \frac{7}{3}}{n^2-1}$$

Which is less than 1.8 for finite values of n and tends to 1.8 as $n \to \infty$

So the measure of skewness $\gamma_1 = 0$

And the measure of kurtosis $\gamma_2 = \beta_2 - 3 < 0$

Thus the Uniform distribution is symmetric and platykurtic.

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