STUDY MATERIAL - 2 (PART - I)

Subject : PHYSICS

Date : 12.5.20

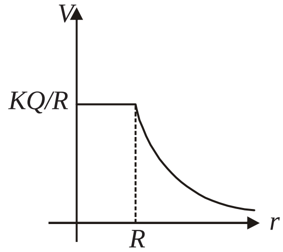
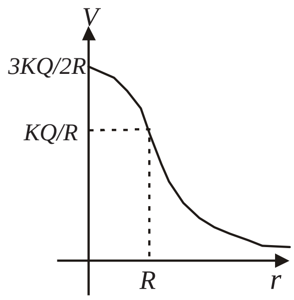
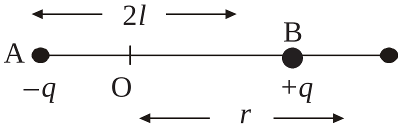
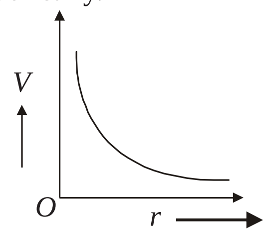
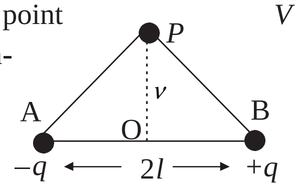
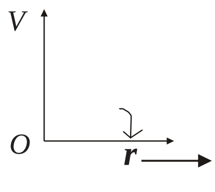
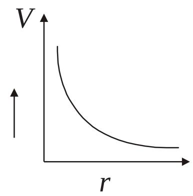
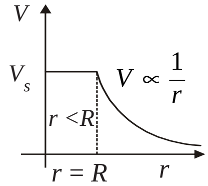
CLASS : XII

Chapter : Electrostatics

Topic : Torque on dipole, potential, potential energy, equipotential surface, potential of a charged spherical conductor

A : Important formulae concepts, diagrams and explanation :

Electric potential due to various charge distributions are given in the Table :

Name/Type	Formula	Note	Graph
1. Uniformly charged hollow conducting/ non-conducting/ solid conducting sphere	for $r \leq R$ $V = \frac{kQ}{R}$ for $r \geq R$ $V = \frac{kQ}{r}$	● R is radius of sphere. ● r is the distance from centre of sphere to the point ● Q is total charge $= \sigma 4 \pi R^2$	
2. Uniformly charged solid non-conducting sphere (insulating material)	for $r \geq R$ $V = \frac{kQ}{r}$ for $r \leq R$ $\frac{KQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\epsilon_0}(3R^2 - r^2)$	● R is radius of sphere. ● r is distance from centre to the point ● $V_{centre} = \frac{3}{2}V_{surface}$ ● Q is total charge $= \rho \frac{4}{3}\pi R^3$ ● Inside sphere potential varies parabolically. ● Outside potential varies hyperbolically.	
3. Potential at a point on the axis of a dipole 	$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{P}{r^2}$ $\therefore \quad V \propto \frac{1}{r^2}$	● For short dipole where $r >> l$	
4. Potential at a point on the perpendicular bisector of a dipole 	$V = O$	● For short dipole where $r >> l$	
5. Potential at a point due to a point charge	$V = \frac{Kq}{r}$	● Distance r from the point charge.	
6. Uniformly charge spherical shell (hollow or solid)	$V_i = V_s = \frac{Kq}{r}$ $= \frac{\sigma R}{\epsilon_o}$ $V_o = \frac{Kq}{r}$		

B : Solved numerical Problems

1. If 100 J of work has to be done in moving an electric charge 4C from a place where potential is –10V to another place where potential is V volt, find the value of V.

   Ans. Here, $W_{AB} = 100\ J,\ q_o = 4C$

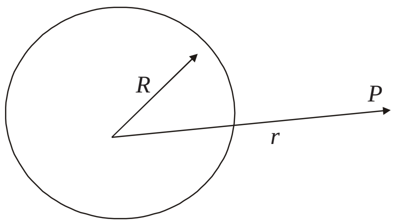
   $V_A = -10\ V,\ V_B = V$

   Since, $V_B - V_A = \frac{W_{AB}}{q_o}$, by external force $V - (-10V) = \frac{100J}{4C} = 25V$

   or $V = 25V - 10\ V = 15\ V$

2. A hollow metal sphere of radius 10 cm is charged such that the potential on its surface becomes 80V. The potential at the centre of the sphere is—

   Ans. The potential at the centre of the sphere is 80V because it remains same at each point under the metallic hollow sphere as on surface.



$$V(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r}; & r>R \\ \frac{Q}{4\pi\varepsilon_0 R}; & r\le R \end{cases}$$

3. A charge 2Q is placed at each corner of a cube of side a. The potential at the centre of the cube is—

   Ans. Let V_1 be the potential at the centre of the cube due to one charge

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{x} \quad \text{and} \quad x = \frac{a\sqrt{3}}{2}$$

   Potential due to all eight corners of the cube $\Rightarrow V = 8V_1 = 8\left[\frac{1}{4\pi\varepsilon_0}\frac{4Q}{\sqrt{3}a}\right] = \frac{32Q}{4\pi\varepsilon_0\sqrt{3}a} = \frac{8Q}{a\sqrt{3}\pi\varepsilon_0}$

4. The potential of a large liquid drop when eight liquid drops are combined is 20V. Then, the potential of each single drop was —

   Ans. Volume of eight drops = Volume of a big drop $\left(\frac{4}{3}\pi r^3\right)\times 8 = \frac{4}{3}\pi R^3 \Rightarrow 2r = R$ ....(i)

   According to charge conservation, $8q = Q$ .... (ii)

   Potential of one small drop $(V') = \frac{q}{4\pi\varepsilon_0 r}$

   Similarly, potential of big drop $(V) = \frac{Q}{4\pi\varepsilon_0 R}$

   Now, $\frac{V'}{V} = \frac{q}{Q}\times\frac{R}{r}$

   $\Rightarrow \frac{V'}{20} = \frac{q}{8q}\times\frac{2r}{r}$ [from Eqs. (i) and (ii)] $\therefore V' = 5$V

5. Determine the electrostatic potential energy of a system consisting of two charges $7\mu C$ and $-2\mu C$ (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively).

   Ans. The electrostatic potential energy due to system of two charges is given as $U = \frac{Kq_1\ q_2}{r_{12}}$

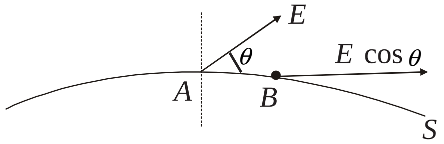
$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = 9\times10^9\times\frac{7\times(-2)\times10^{-12}}{0.18} = -0.7J$$

C : Solution of previous years questions

1. What is an equipotential surface? Show that the electric field is always normal to an equipotential surface. (2016)

Ans. : A surface containing points at the same potential in an electric field is called an equipotential surface.

Let us consider two points A and B very close to each other on an equipotential surface S.



Let the electric field intensity $\boldsymbol{E}$, in the region AB, make an angle θ with the equipotential surface. Since A and B are very close to each other, AB may be taken as a straight line. Component of electric intensity E along $AB = E\cos\theta$.

So work done to move a unit positive charge from A to $B = E\cos\theta \times AB$.

From the property of an equipotential surface, we know that no work is done in moving a charge from one point to another on an equipotential surface.

$\therefore$ $E\cos\theta \times AB = 0$ or, $\cos\theta = 0$ $[\because\ AB \neq 0;\ E \neq 0]$ or, $\theta = \dfrac{\pi}{2}$

2. 64 tiny drops of water having same radius and same charge are combined to form one large drop. The ratio of potential of the large drop. The ratio of potential of the large drop to the small drop is—

Ans. Potential of one tiny drop, $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$. Now, charge of the large water drop = $64q$.

Ratio of the volume of the tiny water drop to the volume of the large water drop = 1 : 64

Then, ratio of their radii = $1 : \sqrt[3]{64} = 1 : 4$

$\therefore$ Potential of the large water drop,

$$V_2 = \frac{1}{4\pi\epsilon_0}\frac{64q}{4r} = 16V_1, \qquad \text{i.e., } \frac{V_2}{V_1} = \frac{16}{1} = 16 : 1$$

3. How much work is done in moving qC of charge between two points separated by a distance r on an equipotential surface? (2013)

Ans. No work has to be done to move q C charge from one point to another on an equipotential surface.

4. Determine how much work is to be done to move a 10C positive charge 1 m along the y-axis in a uniform electric field $\vec{E} = 5(\hat{i} + \hat{j})V\ m^{-1}$ (2014)

Ans. Force acting on a charge q placed in a uniform electric field E is given by,: $\vec{F} = q\vec{E} = 10 \times 5(\hat{i} + \hat{j})N$

Again, displacement along y-axis, $\vec{s} = 1\hat{j}\ m$

$\therefore$ Work done = $\vec{F}.\vec{s} = 50(\hat{i} + \hat{j}).\hat{j} = 50J$

5. A particle of mass M and charge q, initially at rest, is accelerated by a uniform electric field E through a distance D and is then allowed to approach a fixed static charge Q of the same sign. The distance of the closest approach of the charge q will then be (2013)

Ans. Potential energy, $U = qE.D$

At the distance of the closest approach, $\dfrac{Qq}{4\pi\epsilon_0 r} = qED \quad r = \dfrac{Q}{4\pi\epsilon_0 ED}$

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