ST. LAWRENCE HIGH SCHOOL

TOPIC-Linear Simultaneous Equations

Sub: Mathematics **Class-9**

STUDY MATERIAL -3 **Date: 6.5.2020**

Definitions :

1. Simultaneous Linear Equations:
The general form of simple Equation in two unknown is :
$ax + by + c = 0$ (where a and b not equal to 0)

2. If a pair of definite values of two unknown quantities satisfies simultaneously two distinct linear equations in two variables, then those two equations are called simultaneous equations in two variables.
For example : $2x + 3y = 5$ and $4x + 2y = 6$.

3. The graph of linear equation $ax + by + c = 0$ is always a straight line.

4. Every linear equation in two variables has an infinite number of solutions. Here, we will learn about two linear equations in 2 variables. (Both equations having to same variable i.e., x, y)

5. The solution of system of simultaneous linear equation is the ordered pair (x, y) which satisfies both the linear equations.

6. Necessary steps for forming and solving simultaneous linear equations :

Let us take a mathematical problem to indicate the necessary steps for forming simultaneous equations:

In a stationery shop, cost of 3 pencil cutters exceeds the price of 2 pens by Rs2. Also, total price of 7 pencil cutters and 3 pens is Rs43.

Follow the steps of instruction along with the method of solution.

Step I: Indentify the unknown variables; assume one of them as x and the other as y Here two unknown quantities (variables) are:
Price of each pencil cutter = Rsx
Price of each pen = Rsy.

Step II: Identify the relation between the unknown quantities. Price of 3 pencil cutter =Rs3x
Price of 2 pens = Rs2y
Therefore, first condition gives: $3x - 2y = 2$

Step III: Express the conditions of the problem in terms of x and y Again price of 7 pencil cutters =Rs7x
Price of 3 pens = Rs3y
Therefore, second condition gives: $7x + 3y = 43$
Simultaneous equations formed from the problems:
$3x$? $2y = 2$ ------- (i)
$7x + 3y = 43$--------(ii)

<u>For examples:</u>

(i) $x + y = 12$ and $x - y = 2$ are two linear equation (simultaneous equations). If we take $x = 7$ and $y = 5$, then the two equations are satisfied, so we say (7, 5) is the solution of the given simultaneous linear equations.

(ii) Show that $x = 2$ and $y = 1$ is the solution of the system of linear equation $x + y = 3$ and $2x + 3y = 7$

Put $x = 2$ and $y = 1$ in the equation $x + y = 3$
L.H.S. $= x + y = 2 + 1 = 3$, which is equal to R.H.S.

In 2nd equation, $2x + 3y = 7$, put $x = 2$ and $y = 1$ in L.H.S.
L.H.S. $= 2x + 3y = 2 \times 2 + 3 \times 1 = 4 + 3 = 7$, which is equal to R.H.S. Thus, $x = 2$ and $y = 1$ is the solution of the given system of equations.

7. Conditions of solvability of linear simultaneous equations :

For the simultaneous linear equations $ax + by + c = 0$ and $px + qy + r = 0$

(i) Solution is possible if a/p not equal to b/q. This is because the graphs of the two straight lines intersect each other.

For example : $2x + 3y = 6$ and $3x + 2y = -1$.
Here a/p =2/3 not equal to b/q =3/2.
Therefore the equations are solvable.

(ii) Solution is not possible if a/p is equal to b/q but not equal to c/r. Here the graph of two straight lines are parallel to each other.

For example : $3x + 4y = 7$ and $6x + 8y = 20$.
Here 3/6=4/8 but not equal to 7/20.
Therefore the graph of the two equations are two parallel straight line.

(iii) There will be infinite solutions if a/p =b/q = c/r. Here the graph of two straight lines will coincide.

For example : $5x + 6y = 11$ and $10x + 12y = 22$.
Here 5/10 =6/12 =11/22.
Therefore the equations will have infinite solutions.

<u>Methods of solving simultaneous linear equations:</u>

Method of substitution :

1.Solve:

x + y = 7.......................(i)

3x - 2y = 11................. (ii)

Solution:

The given equations are:
x + y = 7 .... (i)
3x - 2y = 11 ....(ii)

From (i) we get y = 7 ⍰ x
Now, substituting the value of y in equation (ii), we
get; 3x - 2 (7 - x) = 11
or, 3x - 14 + 2x =
11 or, 3x + 2x - 14
= 11 or, 5x - 14 =
11
or, 5x -14 + 14 = 11 + 14 [add 14 in both the sides]
or, 5x = 11 + 14
or, 5x = 25
or, 5x/5 = 25/5 [divide by 5 in both the
sides] or, x = 5

Substituting the value of x in equation (i), we
get; x + y = 7
Put the value of x =
5 or, 5 + y = 7
or, 5 ⍰ 5 + y = 7 ⍰ 5
or, y = 7 ⍰
5 or, y = 2

Therefore, x =5 and y =2 is the solution of the simultaneous equations x + y = 7 and 3x ⍰
2y = 11

2. Solve 2x ⍰ 3y = 1 and

3x ⍰ 4y=1.

Solution:

The given equations are:

2x ⍰ 3y = 1 ....(i)
3x ⍰ 4y = 1 ....(ii)

From equation (i), we
get; 2x = 1 + 3y
or, x = ${}^{1}/_{2}(1 + 3y)$

Substituting the value of x in equation (ii),
we get; or, $3 \times {}^1/_2(1 + 3y) – 4y = 1$
or, ${}^3/_2 + {}^9/_2y - 4y = 1$
or, $(9y – 8y)/2 = 1 -$
${}^3/_2$ or, ${}^1/_2y = (2 –$
$3)/2$
or, ${}^1/_2y = −12−12$
or, $y = −12−12 \times 2121$
or, $y = -1$

Substituting the value of y in

equation (i) 2x ⍰ 3 × (-1) = 1
or, $2x + 3 = 1$
or, $2x = 1 - 3$ or, $2x =$
-2 or, $x = -2/2$
or, $x = -1$

Therefore, $x = -1$ and $y = -1$ is the solution of the simultaneous equation 2x ⍰ 3y = 1 and 3x ⍰ 4y = 1.

Method of Comparison :

Solve:

3. $8x-7y +20=0$
   $3x - 2y =0$

Solution :

$8x - 7y+20=0$ or $x =(7y - 20)/8$------(i)
$3x - 2y =0$ or $x =2y /3$ - - - - - (ii)
Comparing the value of x from (i) and (ii) we get,
$(7y - 20)/8 =2y /3$
or $3(7y - 20) =8(2y)$
or $21y - 60 =16y$
or $5y =60$
or $y =60/5 =12$
Putting $y =12$ in (ii) we get,
$x = (2.12)/3 =24/3 =8$
The required solution is $x= 8$, $y= 12$.

Method of Elimination :

Solve:
4. $3x + 2y =12$-------(i)
   $x +2y =8$--------------(ii)
Solution :
Subtracting (ii) from (i) we get,
$3x - x = 12 - 8$ or $2x = 4$ or $x =2$.

Putting x =2 in (ii) we get,
2 +2y =8 or 2y =8 - 2 or y =6/2 =3.
Therefore the required solution is : x =2, y =3.

Method of Cross-multiplication :

Solve:

5. 6/x + 2/y =5
8/x - 3/y =1.

Solution :
Let 1/x = u and 1/y =v.
Therefore the given equations become :
6u +2v - 5 =0
8u - 3v - 1 =0

By cross multiplication we get,

u/(-2-15) =v/(-40+6) =1/(-18-16)
or u/-17 =v/-34 =1/-34
Therefore, u=-17/-34 =1/2 and v = - 34/-34 =1
Since, u=1/x =1/2, so x =2.
Since, v=1/y =1, so y =1.
The required solution is x=2 and y =1.

6. Word problem

10 years ago the age of father was 7 times the age of son. After 2 years twice the age of father will be 5 times the age of son. What are the present age of father and son?

Solution :

Let the present age of father be x years and that of the son is y years.

Therefore, 10 years ago, father's age was (x - 10) and son's age (y-10).

Again, after 2 years, father's age will be (x +2) and son's age (y +2).

From the first condition, (x-10) =7(y-10) or x - 7y = - 60-----(i)

From the second condition, 2(x +2) =5(y +2) or 2x - 5y =6---(ii)

Solving equations (i) and (ii) we get, $x = 38$ and $y = 14$.

Hence the present age of father is 38 years and the present age of son is 14 years.

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