STUDY MATERIAL-11

SUBJECT – MATHEMATICS

1st term

Chapter : Sequence & Series **Class : XI**

Topic: Sums of n terms of an AP **Date: 03.07.2020**

Sums of n terms of an AP :-

Let a be the first term, d the common difference and l the last term of the given AP. If S_n is the sum of n terms, then

$$S_n = a + (a+d) + (a+2d) + \cdots + (l-2d) + (l-d) + l$$

Re-writing S_n in the reverse order, we get

$$S_n = l + (l-d) + (l-2d) + \cdots + (a+2d) + (a+d) + a$$

Adding columnwise, we get

$$2S_n = (a+l) + (a+l) + (a+l) + \cdots + n \text{ times} = n(a+l)$$

$$\Rightarrow S_n = \frac{n}{2}(a+l) = \frac{n}{2}[a+a+(n-1)d] \quad [\text{since } l = a+(n-1)d]$$

Therefore,

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

If S_n is the sum of n terms of an AP whose first term is a and the last term is l, then

$$S_n = \frac{n}{2}(a+l)$$

If S_n is the sum of first n terms of an AP whose last term is l and the common difference is d, then

$$S_n = \frac{n}{2}\{a+l\} = \frac{n}{2}\{l-(n-1)d+l\} = \frac{n}{2}\{2l-(n-1)d\}$$

If a is the first term and d is the common difference of the AP, then the n^{th} term a_n is given by

$$a_n = a + (n-1)d$$

The sum S_n of the first n terms of such an AP is given by

$$S_n = \frac{n}{2}(2a+(n-1)d) = \frac{n}{2}(a+l)$$

where l is the last term.

Examples :-

Example 1. Find the sum of n terms of a series whose 7^{th} term is 30 and 13^{th} term is 54. Hence, or otherwise, find the sum of r terms and 50 terms of the series. Assume the series is in AP.

Solution: Let the first term of AP be a and the common difference be d. The n^{th} term is

$$T_n = a + (n-1)d$$

So the 7^{th} and 13^{th} terms are

$$T_7 = a + 6d = 30 \quad (1)$$

$$T_{13} = a + 12d = 54 \quad (2)$$

Solving Eqs. (1) and (2), we get $a = 6$ and $d = 4$.

Now using

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

we get
$$S_n = \frac{n}{2}[2 \times 6 + (n-1) \times 4] = 2n(n+2)$$

Now find the sum of r terms, S_r. Using $S_n = 2n(n+2)$, we replace n by r to get S_r.

$$S_r = 2r(r+2)$$

To find the sum of 50 terms, we can use

$$S_n = 2n(n+2)$$

$$\Rightarrow S_{50} = 2 \times 50(50+2) = 5200$$

Example 2. The sum of n terms of two series in AP is in the ratio $5n+4:9n+6$. Find the ratio of their 13th term.

Solution: Let a_1 and a_2 be the first terms of two APs and d_1 and d_2 be their respective common differences. Then

$$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]}=\frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2}=\frac{5n+4}{9n+6} \qquad (1)$$

Now the ratio of 13th terms $=\dfrac{a_1+12d_2}{a_2+12d_2}$

Comparing this with LHS of Eq. (1) we get

$$\frac{n-1}{2}=12\Rightarrow n=25$$

Hence, we have

$$\Rightarrow \frac{a_1+12d_1}{a_2+12d_2}=\frac{5(25)+4}{9(25)+6}=\frac{129}{231}$$

Example 3. The sum of first p, q and r terms of an AP is a, b and c, respectively. Show that

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

Solution: Let A be the first term and D be the common difference of the AP. Then

$$\Rightarrow\ a=\frac{p}{2}[2A+(p-1)D]$$

We can write

$$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=\sum\frac{a}{p}(q-r)$$

Now

$$\text{LHS}=\sum\frac{a}{p}(q-r)$$

$$=\sum\frac{1}{2}(q-r)[2A+(p-1)D]$$

$$=\frac{1}{2}\sum 2A(q-r)+\frac{1}{2}\sum(q-r)D(p-1)$$

$$=A\sum(q-r)+\frac{D}{2}\sum[p(q-r)]-\frac{D}{2}\sum(q-r)$$

$$=0+0-0=0=\text{RHS}$$

Example 4. The sum of n, $2n$ and $3n$ terms of an AP is S_1, S_2 and S_3, respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Solution: The S_1, S_2, S_3 terms can be written as

$$S_1 = \frac{n}{2}[2a(n-1)d]$$

$$S_2 = \left(\frac{2n}{2}[2a+(2n-1)d]\right)$$

$$S_3 = \frac{3n}{2}[2a+(3n-1)d]$$

Now

$$S_2 - S_1 = \frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[2a+(3n-1)d]$$

$$3(S_2 - S_1) = \frac{3n}{2}[2a+(3n-1)d] = S_3$$

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