ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

Sub: Algebra and Geometry **Class: 7** **Date: 04.05.20**

STUDY MATERIAL: PARALLEL LINES

Concepts

What are Parallel Lines?

Two lines are said to be parallel when they do not meet at any point in a plane. Lines which do not have a common intersection point and never cross path with each other are parallel to each other. The symbol for showing parallel lines is ||. Two lines which are parallel are represented as $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, which means that the line $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.The perpendicular distance between the two parallel lines is always constant.

Figure 1: Parallel Lines

In the figure shown above, the line segment $\overline{PQ}$ and $\overline{RS}$ represent two parallel lines as they have no common intersection point in the given plane. Infinite parallel lines can be drawn parallel to $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ in the given plane.
Lines can either be parallel or intersecting. When two lines meet at a point in a plane, they are known as intersecting lines. If a line intersects two or more lines at distinct points then it is known as a transversal line.

In fig. 2, the line l intersects the lines a and b at points P and Q respectively. The line l is the transversal here.
∠1,∠2,∠7 and ∠8 are the exterior angles and ∠3,∠4,∠5 and ∠6 denote the interior angles. The angle pairs formed due to intersection by transversal are named as follows:

1. Corresponding Angles: ∠1 and ∠6; ∠4 and ∠8; ∠2 and ∠5; ∠3 and ∠7 are the corresponding pair of angles.
2. Alternate Interior Angles: ∠4 and ∠5 ; ∠3 and ∠6 denote the pair of alternate interior angles.
3. Alternate Exterior Angles: ∠1 and ∠7; ∠2 and ∠8 are the alternate exterior angles.
4. Same side Interior Angles: ∠3 and ∠5; ∠4 and ∠6 denote the interior angles on the same side of the transversal or co-interior or consecutive interior angles.

Nilanjana Bhadra

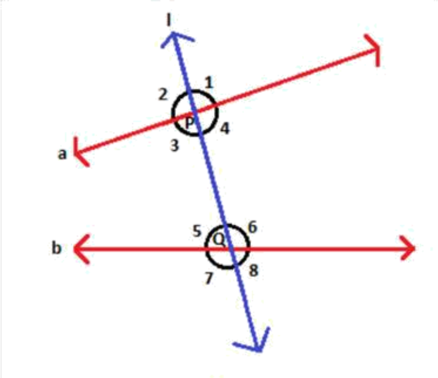


Figure 2 Lines and Transversal

If the lines a and b are parallel to each other as shown, then the following axioms are given for angle pairs of these lines.

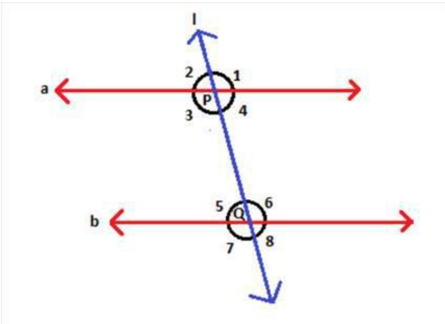


Figure 3 Parallel Lines

Parallel Lines Axioms and Theorems

Go through the following axioms and theorems for the parallel lines.

Corresponding Angle Axiom

If two lines which are parallel are intersected by a transversal then the pair of corresponding angles are equal.
From Fig. 3: $\angle1=\angle6$, $\angle4=\angle8$, $\angle2= \angle5$ and $\angle3= \angle7$

The converse of this axiom is also true according to which if pair of corresponding angles are equal then the given lines are parallel to each other.

Theorem 1

If two lines which are parallel are intersected by a transversal then the pair of alternate interior angles is equal.
From Fig. 3: $\angle4=\angle5$ and $\angle3=\angle6$
Proof: As, $\angle4=\angle2$ and $\angle1=\angle3$(Vertically Opposite Angles)
Also, $\angle2=\angle5$ and $\angle1=\angle6$ (Corresponding Angles)
$\Rightarrow\angle4=\angle5$ and $\angle3=\angle6$

The converse of the above theorem is also true which states that if the pair of alternate interior angles are equal then the given lines are parallel to each other.

Theorem 2

If two lines which are parallel are intersected by a transversal then the pair of interior angles on the same side of transversal are supplementary.
∠3+ ∠5=180° and ∠4+∠6=180°
As ∠4=∠5 and ∠3=∠6 (Alternate interior angles)
∠3+ ∠4=180° and ∠5+∠6=180° (Linear pair axiom)
⇒∠3+ ∠5=180° and ∠4+∠6=180°

The converse of the above theorem is also true which states that if the pair of co-interior angles are supplementary then the given lines are parallel to each other.

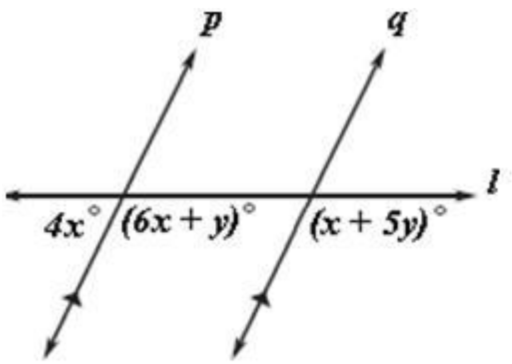
Applications of Parallel Lines in Real Life

One will be able to see lines which are parallel to each other in real life too, if only one has the patience and is observant enough to do so. For instance take the railroads. The railway tracks are literally parallel lines. The two lines or tracks are meant for the wheels of the train to travel along on. The difference between the parallel lines imagined by mathematicians and the ones who actually make the railway tracks is that, mathematicians have the liberty to imagine the parallel lines over flat surfaces and paper, while trains travel across all sorts of terrain, from hills, slopes and mountains to over bridges.

According to mathematicians when two parallel lines are graphed, they must always be at the same angle, which means they'll have the same slope or steepness.

Solved Numericals

Question: In the given figure, p || q and l is a transversal. Find the values of x and y.



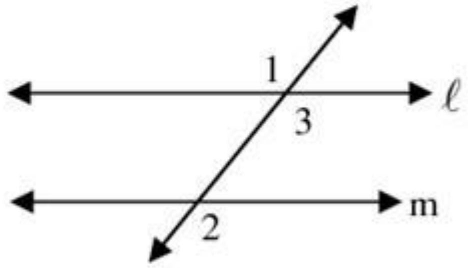
Solution: $6x+y=x+5y6x+y=x+5y$ (Corresponding angles are equal)
Or, $6x–x=5y–y6x–x=5y–y$
Or, $5x=4y5x=4y$
Or, $x=4y5x=4y5$

Now, $4x+6x+y=180°4x+6x+y=180°$ (linear angles are supplementary)
Or, $10x+y=180°10x+y=180°$
Or, $40y5+y=180°40y5+y=180°$
Or, $45y5=180°45y5=180°$
Or, $45y=180°×5=900°45y=180°×5=900°$

Or, y=20°y=20°

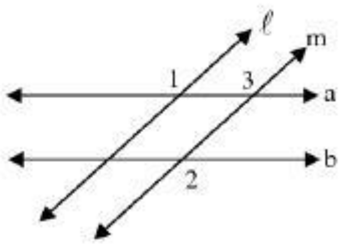
Hence, x=4×205=16°x=4×205=16°

Question: In the given figure, angle 1 = angle 2, then prove l||m.



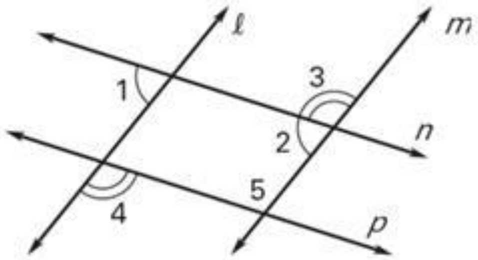
Solution: Construction: A transversal intersects two parallel lines on two distinct points.
Given: ∠ 1 = ∠ 2
To prove l||m
Proof: ∠ 1 = ∠ 2 (Given)
∠ 1 = ∠ 3 (Vertically opposite angles are equal)
From above equations, it is clear;
∠ 3=∠2
Since corresponding angles are equal hence, l||m proved.

Question: In the given figure, ;||m and angle 1 = angle 2, then prove that a||b.



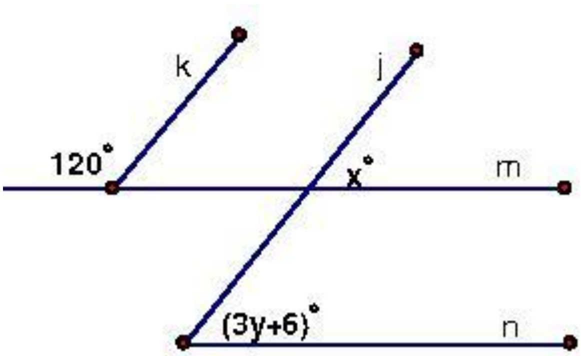
Solution: Construction: Line l||m which are intersected by two transversals a and b.
Given ∠1= ∠2
To prove a||b
Let us name the angle which is vertically opposite to ∠ 3, as ∠ 4.
Proof: ∠ 1 = ∠ 2 (Given)
∠ 1 = ∠ 3 (corresponding angles are equal)
From above equations, it is clear:
∠ 2=∠3
∠ 3 = ∠ 4 (Vertically opposite angles are equal.
From above equations, it is clear:

$\angle 2 = \angle 4$
Since corresponding angles are equal hence, a||b proved.
Question: In the given figure, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, then prove l||m and n||p



Solution: Construction: Lines l and m are intersected by transversals n and p at distinct points.
Given; $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$
To prove l||m and n||p
Proof: : $\angle 1 = \angle 2$ (given)
Since these are corresponding angles and are equal, so l||m is proved. Now,
$\angle 3 = \angle 4$ (given)
Since these are alternate interior angles, so n||p is proved.

Question: In the given figure, k||j and m||n, then find the values of x and y.

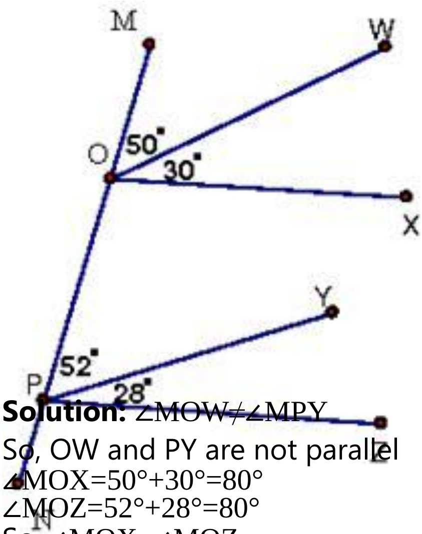


Solution: Let us name the angle adjacent to 120° as z.
$120° + z = 180°$ (Linear pair of angles is supplementary) Or,
$z = 180° - 120° = 60°$
$\angle x = \angle z = 60°$ (Corresponding angles are equal)

Now;
$\angle x = \angle(3y+6)$ (Corresponding angles are equal)
Or, $3y + 6 = 60°$
Or, $3y = 60° - 6 = 54°$

Or, $y=54\div 3=18^\circ$
Hence, $x=60^\circ$ and $y=18^\circ$

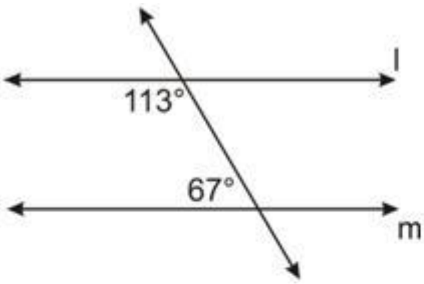
Question: In the following figure, find the pair of parallel lines.



Solution: $\angle MOW \neq \angle MPY$
So, OW and PY are not parallel
$\angle MOX=50^\circ+30^\circ=80^\circ$
$\angle MOZ=52^\circ+28^\circ=80^\circ$
So, $\angle MOX=\angle MOZ$

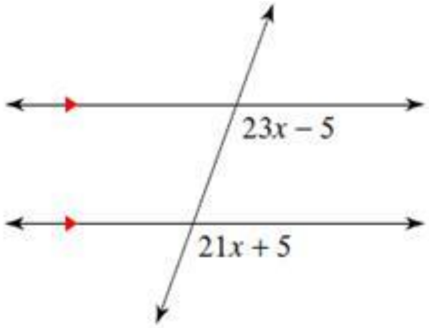
Since corresponding angles are equal, so OX||OZ
Hence, OX||PZ

Question: In the following figure, a transversal is intersecting two lines at distinct points. Prove l||m.



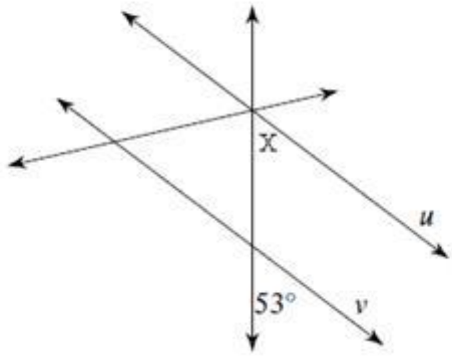
Solution: $\angle$ 113° + 67° = 180°
Since internal angles on the same side of transversal are supplementary,
Hence, l||m proved.

Question: In the given figure, a transversal is intersecting two parallel lines at distinct points. Find the value of x.



Solution: $23x–5=21x+5$ (Corresponding angles are equal)
Or, $23x=21x+10$
Or, $23x–21x=10$
Or, $2x=10$
Or, $x=5$

Question: If u and v are parallel lines, find the value of x.

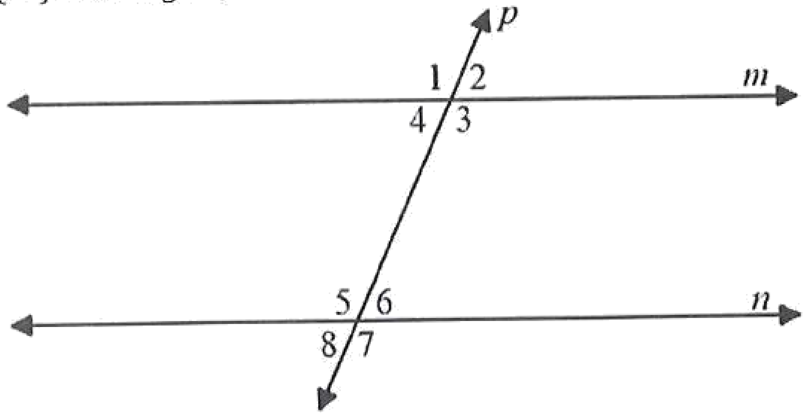


Solution: Since corresponding angles are equal
Hence, $x=53^0$

Previous Years Solution
2019
1st Term

(iv) If two lines are parallel than the alternate interior angles are:
(a) equal; (b) not equal; (c) complementary; (d) supplementary.
Ans: (a) equal
(v) Mention three conditions to prove two lines are parallel.
Ans: Two lines are parallel, if :
a) a pair of corresponding angles are equal
b) alternate interior angles are equal.
c) the same side interior angles are supplementary.

(vii) In the figure, if $\angle 1 = 110^0$ and $\angle 6 = 70^0$, show that line $m \| n$.



Ans: $\angle 1 + \angle 2 = 180^0$

Or $110^0 + \angle 2 = 180^0$

Or $\angle 2 = 180^0 - 110^0 = 70^0$

Given $\angle 6 = 70^0$.

$\angle 2$ and $\angle 6$ are corresponding angles and $\angle 2 = \angle 6$ so $m \| n$.

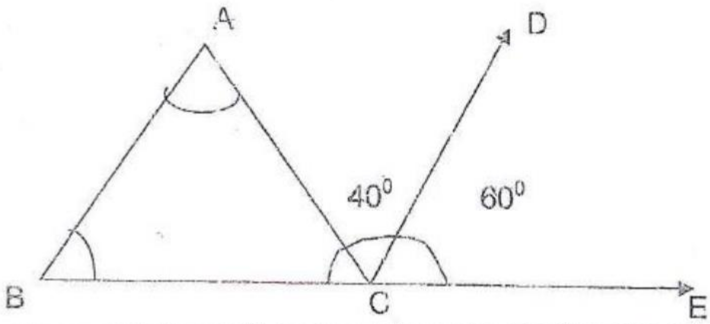
2018

1st Term

(v) Conditions for proving two lines are parallel:

- The alternate angles are equal.
- The corresponding angles are equal.
- The sum of two co-interior angles is 180^0

(vii) In the given figure, $\angle DCE = 60^0$ and $\angle ACD = 40^0$, Find $\angle A$, $\angle B$ and $\angle C$, when AB ǁ CD.



(vii) $\angle A = \angle ACD = 40^0$, being alternate angles are equal.

$\angle B = \angle DCE = 60^0$, being corresponding angles are equal.

$\angle C = 80^0$, since sum of three angles of a triangle is 180^0.

Exercise Problems

Question 1

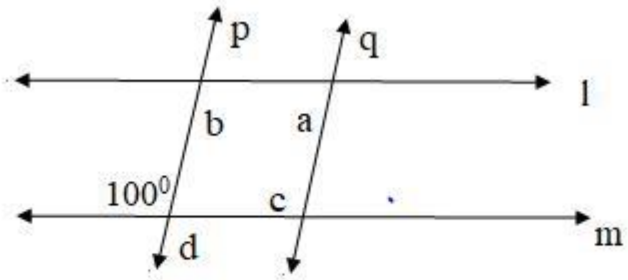
fill in the blanks:

a) Two ______________ are said to form linear pair of the angels if their non-common arms are two opposite rays.

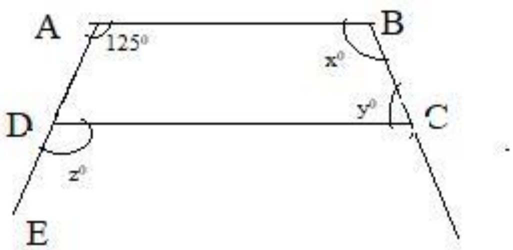
b) if a ray stands on a line then the sum of the adjacent angles so formed is

_______________.
c) The sum of all angles around a point is _________________.
d) An angle which is equal to its complement is _____________.
e) Two angles are called a pair of _____________________ if their arms form two pairs of opposite rays.
f) If two lines intersect then the Vertically opposite angles are _____________
g) A line which intersects two or more lines at distinct points is called a ___________________
h) You can draw _____________ transversals on a line.
(i) If two lines are intersected by a transversal such that any pair of corresponding angles are equal then the lines are _________________

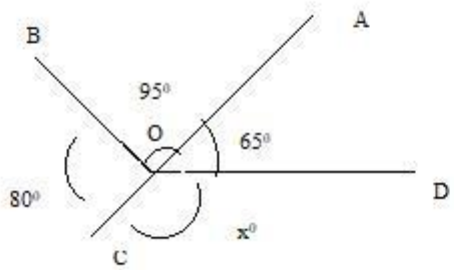
Question 2
Two parallel lines I and m are intersected by a transversal t. If the interior angles on same side of transversal are $(2x-8)^{o}$ and $(3x-7)^{o}$ Find the measure of these angles.
Question 3
In the given fig. I is parallel to m and p ll q. Find the measure of each of the angles a,b,c,d.



Question 4
In the given figure below. AB II CD and AD is produced to E so that $\angle BAE=125^{o}$.
if $\angle ABC=x^{o}$, $\angle BCD=y^{o}$ and $\angle CDE=z^{o}$ and $\angle ADC=x^{o}$ Find the values of x,y and z



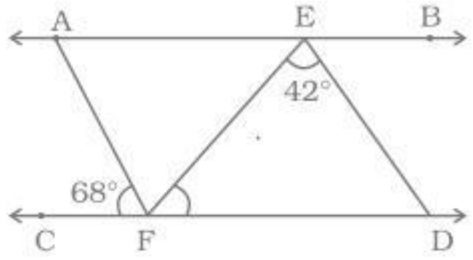
Question 5
In the given below figure rays OA,OB,OC and OD intersect at a point. Find the value of x



Question 6
True and False statement
a. Sum of two complementary angles is 180°.
b.Sum of two supplementary angles is 180°.
c. Sum of interior angles on the same side of a transversal with two parallel lines is 90°.

d. Vertically opposite angles are equal.
e. A linear pair may have two acute angles
f. Two supplementary angles are always obtuse angles

Question 7
In below figure , AB||CD, AF||ED, $\angle AFC=68^o$ and $\angle FED=42^o$ Find $\angle EFD$



Question 8
In below figure, l ||m ||n. ∠ QPS = 35° and ∠ QRT = 55°. Find ∠PQR