ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-15

SUBJECT – STATISTICS

Pre-test

Chapter: THEORITICAL PROBABILITY DISTRIBUTION **Class: XII**

Topic: POISSON PROBABILITY DISTRIBUTION **Date: 27.06.20**

PROBABILITY DISTRIBUTION

PART 9

A random variable X follows Poisson distribution with parameter λ

$$X \sim \text{poissn}(\lambda)$$

The pmf of the random variable X is given by

$$f(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}, x = 0(1)\infty$$

1. If $P(X=3) = P(X=4)$ for a Poisson random variable X, then find

(i) the mean of the distribution, (ii) P(X=0), (iii) $P(1 \leq X \leq 3)$.

Solution:

$$P(X=3) = P(X=4)$$

$$\Rightarrow \frac{e^{-\lambda} \lambda^{3}}{3!} = \frac{e^{-\lambda} \lambda^{4}}{4!}$$

$$\Rightarrow \lambda = 4$$

(i) So the mean of the distribution is 4.

(ii) P(X=0) = $\frac{e^{-4} 4^{0}}{0!} = e^{-4}$

(iii) $P(1 \leq X \leq 3) = \frac{e^{-4} 4^{1}}{1!} + \frac{e^{-4} 4^{2}}{2!} + \frac{e^{-4} 4^{3}}{3!} = \frac{68}{3e^{4}}$

2. If a random variable X follows Poisson distribution satisfying P(X=0)=P(X=1), determine the value of $P(X > 1 | X < 3)$ and expectation of X.

Solution:

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

P(X=0)=P(X=1)

$$\Rightarrow \frac{e^{-\lambda} \lambda^0}{0!} = \frac{e^{-\lambda} \lambda^1}{1!}$$

$$\Rightarrow \lambda = 1$$

$$P(X > 1 \, I \, X < 3)$$

$$= \frac{P\ (1 < X < 3)}{P(X < 3)}$$

$$= \frac{P\ (X = 2)}{P(X < 3)}$$

$$= \frac{\frac{1}{2}}{\frac{5}{2}} = \frac{1}{5}$$

3. A discrete random variable X follows Poisson distribution, find the values of

(i) P(X=at most1), (ii) P(X>1|X>0). [Given E(X)=2.5 and $e^{-2.5} = 0.082$]

Solution:

(i) P(X=at most1) = P (X=0) + P(X=1)

$$= \frac{e^{-2.5}}{0!} + \frac{e^{-2.5}*2.5}{1!}$$

$$= 0.287$$

(ii) $P(X>1|X>0) = \frac{P(X>1)}{P(X>0)} = \frac{1-(f(0)+f(1))}{1-f(0)}$

$$= \frac{0.713}{0.918} = 0.7767$$

4. Show that the function

$$f(x) = \begin{cases} \frac{e^{-m} m^x}{(1-e^{-m}).x!}, 0 < m < \infty, (x = 1,2,\dots,\infty) \\ 0 \qquad , otherwise \end{cases}$$

represents a p.m.f.

Solution:

$$f(x) = \frac{e^{-m} m^x}{(1-e^{-m})x!}\ ,\ x = 1(1)\infty$$

$e^{-m} > 0\ , m^x > 0\ \ since\ 0 < m < \infty, x! > 0\ since\ x > 0$

So $f(x) > 0$

$$\sum_{x=1}^{\infty} f(x) = 1$$

$$= \sum_{x=1}^{\infty} \frac{e^{-m} m^x}{(1-e^{-m})x!}$$

$$= \frac{e^{-m}}{(1-e^{-m})} \sum_{x=1}^{\infty} \frac{m^x}{x!}$$

$$= \frac{e^{-m}}{(1-e^{-m})} (e^m - 1) = 1$$

Hence f(x) satisfies both the conditions of a pmf.

5. Determine f(x), the p.m.f., from $f(x) = \frac{\lambda}{x} f(x-1), x = 1,2,3,..$, where f(x) is non-zero for non-negative integral values of the random variable x. Find also the probability that X is greater than zero.

Solution:

$$f(x) = \frac{\lambda}{x} f(x-1)$$

$$f(x) = \frac{\lambda}{x} \frac{\lambda}{(x-1)} f(x-2)$$

$$f(x) = \frac{\lambda}{x} \frac{\lambda}{(x-1)} \frac{\lambda}{(x-2)} f(x-3)$$

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$$f(x) = \frac{\lambda}{x}\,\frac{\lambda}{(x-1)}\,\frac{\lambda}{(x-2)}\,\frac{\lambda}{(x-3)}\,\ldots\ldots\frac{\lambda}{(x-(x-1))}\,f(x-x)$$

$$\sum_{x=1}^{\infty} f(x) = 1$$

$$\Rightarrow \sum_{x=1}^{\infty} f(x) = 1 \quad \Rightarrow \sum_{x=1}^{\infty} \frac{\lambda^x}{x!}\,f(0) = 1$$

$$\Rightarrow f(0) \sum_{x=1}^{\infty} \frac{\lambda^x}{x!} = 1$$

$$\Rightarrow f(0) = \frac{1}{(1-e^{\lambda})}$$

So $f(x) = \frac{1}{(1-e^{\lambda})}\frac{\lambda^x}{x!}, \qquad x = 1(1)\infty$

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