ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-12

SUBJECT – MATHEMATICS

Pre-test

Chapter: Differentiation **Class: XII**

Topic: Differentiation **Date: 17.06.2020**

-: Differentiation (Part I) :-

Differentiation techniques

In this section we will discuss about different techniques to obtain the derivatives of the given functions.

1. Some standard results [formulae]

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(x) = 1$
3. $\frac{d}{dx}(k) = 0$, k is a constant
4. $\frac{d}{dx}(kx) = k$, k is a constant
5. $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$
6. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
7. $\frac{d}{dx}(e^x) = e^x$
8. $\frac{d}{dx}(e^{ax}) = ae^{ax}$
9. $\frac{d}{dx}(e^{ax+b}) = ae^{ax+b}$
10. $\frac{d}{dx}(e^{-x^2}) = -2x\,e^{-x^2}$
11. $\frac{d}{dx}(a^x) = a^x \log_e a$
12. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$
13. $\frac{d}{dx}\log_e(x+a) = \frac{1}{x+a}$
14. $\frac{d}{dx}(\log_e(ax+b)) = \frac{a}{(ax+b)}$
15. $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$
16. $\frac{d}{dx}(\sin x) = \cos x$
17. $\frac{d}{dx}(\cos x) = -\sin x$
18. $\frac{d}{dx}(\tan x) = \sec^2 x$
19. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
20. $\frac{d}{dx}(\sec x) = \sec x \tan x$
21. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

2. General rules for differentiation

(i) Addition rule

$$\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$$

(ii) Subtraction rule (or) Difference rule

$$\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)]$$

(iii) Product rule

$$\frac{d}{dx}[f(x)\cdot g(x)]=\frac{d}{dx}[f(x)]g(x)+f(x)\frac{d}{dx}[g(x)]$$

(or) If $u=f(x)$ and $v=g(x)$ then, $\frac{d}{dx}(uv)=u\frac{d}{dx}(v)+v\frac{d}{dx}(u)$

(iv) Quotient rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

(or) If $u=f(x)$ and $v=g(x)$ then, $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{d}{dx}(u)-u\frac{d}{dx}(v)}{v^2}$

(v) Scalar product

$\frac{d}{dx}[cf(x)]=c\frac{d}{dx}[f(x)]$, where c is a constant.

(vi) Chain rule:

$$\frac{d}{dx}[f(g(x))]=f'(g(x))\cdot g'(x)$$

(or) If $y=f(t)$ and $t=g(x)$ then, $\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$

Differentiation of Inverse Trigonometry Functions

$$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad -1 < x < 1$$

$$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad -1 < x < 1$$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2} \qquad -\infty < x < \infty$$

$$\frac{d(\cot^{-1} x)}{dx} = \frac{-1}{1+x^2} \qquad -\infty < x < \infty$$

$$\frac{d(\sec^{-1} x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}} \qquad |x| > 1$$

$$\frac{d(\text{cosec}^{-1} x)}{dx} = \frac{-1}{x\sqrt{x^2-1}} \qquad |x| > 1$$

Example 1.

Differentiate the following functions with respect to x.

(i) $x^{\frac{3}{2}}$ (ii) $7e^x$ (iii) $\frac{1-3x}{1+3x}$

(iv) $x^2 \sin x$ (v) $\sin^3 x$ (vi) $\sqrt{x^2+x+1}$

Solution

(i) $\frac{d}{dx}\left(x^{\frac{3}{2}}\right) = \frac{3}{2}x^{\frac{3}{2}-1}$

$= \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$

(ii) $\frac{d}{dx}(7e^x) = 7\frac{d}{dx}(e^x) = 7e^x$

(iii) Differentiating $y = \frac{1-3x}{1+3x}$ with respect to x

$$\frac{dy}{dx} = \frac{(1+3x)\frac{d}{dx}(1-3x)-(1-3x)\frac{d}{dx}(1+3x)}{(1+3x)^2}$$

$$= \frac{(1+3x)(-3)-(1-3x)(3)}{(1+3x)^2} = \frac{-6}{(1+3x)^2}$$

(iv) Differentiating $y = x^2 \sin x$ with respect to x

$$\frac{dy}{dx} = x^2\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(x^2)$$

$$= x^2\cos x + 2x\sin x$$

$$= x(x\cos x + 2\sin x)$$

(v) $y = \sin^3 x$ (or) $(\sin x)^3$

Let $u = sin\ x$ then $y = u^3$

Differentiating $u= sin\ x$ with respect to x

We get, $\frac{du}{dx} = \cos x$

Differentiating $y = u^3$ with respect to u

We get, $\frac{dy}{du} = 3u^2 = 3\sin^2 x$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = 3\sin^2 x \cdot \cos x$

(vi) $y = \sqrt{x^2+x+1}$

Let $u = x^2 + x + 1$ then $y = \sqrt{u}$

Differentiating $u = x^2 + x + 1$ with respect to x

We get, $\frac{du}{dx} = 2x + 1$

Differentiating $y = \sqrt{u}$ with respect to u.

We get, $\frac{dy}{du} = \frac{1}{2\sqrt{u}}$

$$= \frac{1}{2\sqrt{x^2+x+1}}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{x^2+x+1}} \cdot (2x+1)$$

$$= \frac{2x+1}{2\sqrt{x^2+x+1}}$$

HOME-WORK

1. Differentiate the following with respect to x.

(i) $3x^4 - 2x^3 + x + 8$ (ii) $\frac{5}{x^4} - \frac{2}{x^3} + \frac{5}{x}$ (iii) $\sqrt{x} + \frac{1}{\sqrt[3]{x}} + e^x$

(iv) $\frac{3+2x-x^2}{x}$ (v) $x^3 e^x$ (vi) $(x^2 - 3x + 2)(x+1)$

(vii) $x^4 - 3\sin x + \cos x$ (viii) $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

2. Differentiate the following with respect to x.

(i) $\frac{e^x}{1+x}$ (ii) $\frac{x^2+x+1}{x^2-x+1}$ (iii) $\frac{e^x}{1+e^x}$

3. Differentiate the following with respect to x.

(i) $x \sin x$ (ii) $e^x \sin x$ (iii) $e^x(x+\log x)$

(iv) $\sin x \cos x$ (v) $x^3 e^x$

4. Differentiate the following with respect to x.

(i) $\sin^2 x$ (ii) $\cos^2 x$ (iii) $\cos^3 x$

(iv) $\sqrt{1+x^2}$ (v) $(ax^2+bx+c)^n$ (vi) $\sin(x^2)$

(vii) $\frac{1}{\sqrt{1+x^2}}$

Prepared by -

Mr. SUKUMAR MANDAL (SkM)