ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-6

SUBJECT – MATHEMATICS

1st term

Chapter: Trigonometry **Class: XI**

Topic: Trigonometric Identities **Date: 24.06.2020**

Trigonometric Ratios and Identities

(Solved MCQs – Set 1) :-

1. If $\tan x = n \cdot \tan y$, $n \in R^+$, then maximum value of $\sec^2(x-y)$ is equal to

(A) $\frac{(n+1)^2}{2n}$ **(B)** $\frac{(n+1)^2}{n}$

(C) $\frac{(n+1)^2}{2}$ **(D)** $\frac{(n+1)^2}{4n}$

2. If $3\sin\theta + 5\cos\theta = 5$, then the value of $5\sin\theta - 3\cos\theta$ is equal to

(A) 5 **(B)** 3

(C) 4 **(D)** None of these

3. In ΔABC, if $\cot A \cdot \cot B \cdot \cot C > 0$, then the triangle is

(A) Acute angled **(B)** Right angled

(C) Obtuse angled **(D)** Does not exist

4. If $\pi < 2\theta < \frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals

(A) $-2\cos\theta$ **(B)** $-2\sin\theta$

(C) $2\cos\theta$ **(D)** $2\sin\theta$

5. If $\tan\theta = \sqrt{n}$ for some non-square natural number n, then $\sec 2\theta$ is

(A) A rational number **(B)** An irrational number

(C) A positive number **(D)** None of these

(Non-square number is a number which is not a perfect square)

6. The minimum value of $\cos(\cos x)$ is

(A) 0 **(B)** $-\cos 1$

(C) $\cos 1$ **(D)** -1

7. If $\sin x \cos y = 1/4$ and $3 \tan x = 4 \tan y$, then find the value of $\sin(x + y)$.

(A) 1/16 **(B)** 7/16

(C) 5/16 **(D)** None of these

8. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is

(A) $4+\sqrt{2}$ **(B)** $3+\sqrt{2}$

(C) 9 **(D)** 4

9. If α and β are solutions of $\sin^2 x + a \sin x + b = 0$ as well as that of $\cos^2 x + c \cos x + d = 0$, then $\sin(\alpha + \beta)$ is equal to

(A) $\frac{2bd}{b^2+d^2}$ **(B)** $\frac{a^2+c^2}{2ac}$

(C) $\frac{b^2+d^2}{2bd}$ **(D)** $\frac{2ac}{a^2+c^2}$

10. If $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in GP, then roots of the equation $x^2 + 2x \cot \beta + 1 = 0$ are always

(A) Equal **(B)** Real

(C) Imaginary **(D)** Greater than 1

11. If $S = \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \cdots + \cos^2\frac{(n-1)\pi}{n}$, then S equals

(A) $\frac{n}{2}(n+1)$ **(B)** $\frac{1}{2}(n-1)$

(C) $\frac{1}{2}(n-2)$ **(D)** $\frac{n}{2}$

12. If in a ΔABC, $\angle C = 90^\circ$, then the maximum value of $\sin A \sin B$ is

(A) $\frac{1}{2}$
(B) 1
(C) 2
(D) None of these

13. If in a ΔABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is always

(A) Isosceles triangle
(B) Right angled
(C) Acute angled
(D) Obtuse angled

14. Maximum value of the expression $2\sin x + 4\cos x + 3$ is

(A) $2\sqrt{5}+3$
(B) $2\sqrt{5}-3$
(C) $\sqrt{5}+3$
(D) None of these

15. If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1-\cos\alpha\cos\beta}$, then one of the values of $\tan\frac{\theta}{2}$ is

(A) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$
(B) $\cot\frac{\alpha}{2}\tan\frac{\beta}{2}$
(C) $\sin\frac{\alpha}{2}\sin\frac{\beta}{2}$
(D) None of these

16. If $0 < \theta < \frac{\pi}{4}$ then $\sec 2\theta - \tan 2\theta$ is equal to

(A) $\tan\left(\frac{\pi}{4}+\theta\right)$
(B) $-\tan\left(\frac{\pi}{4}-\theta\right)$
(C) $\tan\left(\frac{\pi}{4}-\theta\right)$
(D) None of these

17. The equation $(\cos p - 1)\,x^2 + (\cos p)x + \sin p = 0$, where x is a variable, has real roots. Then the interval of p may be any one of the following:

(A) $(0, 2\pi)$ **(B)** $(-\pi, 0)$

(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **(D)** $(0, \pi)$

18. Let n be a positive integer such that $\sin\frac{\pi}{2^n} + \cos\frac{\pi}{2^n} = \frac{\sqrt{n}}{2}$ then

(A) $6 \le n \le 8$ **(B)** $4 \le n \le 8$

(C) $4 < n \le 8$ **(D)** $4 < n < 8$

19. If $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta + \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta - \frac{2\pi}{3}\right)}$, then $x + y + z$ is equal to

(A) -1 **(B)** 1

(C) 0 **(D)** None of these

20. If $A + B + C = 180°$, then the value of $\tan A + \tan B + \tan C$ is

(A) $\ge 3\sqrt{3}$ **(B)** $\ge 2\sqrt{3}$

(C) $> 3\sqrt{3}$ **(D)** $> 2\sqrt{3}$

21. Let $0 < A, B < \frac{\pi}{2}$ satisfying the equalities $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$. Then $A + 2B =$

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ **(D)** None of these

22. If $a\ \cos^3\theta + 3a\ \cos\theta\ \sin^2\theta = x$ and $a\ \sin^3\theta + 3a\ \cos^2\theta\ \sin\theta = y$, then $(x+y)^{2/3} + (x-y)^{2/3} =$

(A) $2a^{2/3}$ **(B)** $a^{2/3}$

(C) $3a^{2/3}$ **(D)** $2a^{1/3}$

23. If $(1+\sqrt{1+a})\tan\alpha = (1+\sqrt{1-a})$, then $\sin 4\alpha =$

(A) $a/2$ **(B)** a

(C) $a^{2/3}$ **(D)** $2a$

-:SOLUTIONS:-

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1. $\cos(x-y) = \cos x \cos y + \sin x \sin y = \cos x \cos y\,(1 + \tan x \tan y)$
Put $\tan x = n \tan y$

$\cos(x-y) = \cos x \cos y(1 + n\tan^2 y) \Rightarrow \sec(x-y) = \dfrac{\sec x \sec y}{(1+n\tan^2 y)}$

$$\sec^2(x-y) = \frac{\sec^2 x \cdot \sec^2 y}{(1+n\tan^2 y)^2}$$

$$= \frac{(1+\tan^2 x)(1+\tan^2 y)}{(1+n\tan^2 y)^2}$$

$$= \frac{(1+n^2\tan^2 y)(1+\tan^2 y)}{(1+n\tan^2 y)^2}$$

$$= 1 + \frac{(n-1)^2\tan^2 y}{(1+n\tan^2 y)^2}$$

(By division) using inequality

$$\frac{(1+n^2\tan^2 y + n^2\tan^2 y + n^4\tan^4 y)}{4} \geq (n^4\tan^8 y)^{1/4}$$

Now,

$$\frac{(1+n\tan^2 y)^2}{4} \geq n\tan^2 y$$

$$\Rightarrow \frac{\tan^2 y}{(1+n\tan^2 y)^2} \leq \frac{1}{4n}$$

$$\Rightarrow \sec^2(x-y) \leq 1 + \frac{(n-1)^2}{4n} = \frac{(n+1)^2}{4n}$$

2. $3\sin\theta = 5(1-\cos\theta) = 5 \times 2\sin^2(\theta/2) \Rightarrow \tan(\theta/2) = 3/5$

$$5\sin\theta - 3\cos\theta = 5 \times \frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)} - 3\frac{[1-\tan^2(\theta/2)]}{1+\tan^2(\theta/2)}$$

$$= 5 \times \frac{2\times(3/5)}{1+(9/25)} - \frac{3\times[1-(9/25)]}{1+(9/25)} = 3$$

3. Since $\cot A \cot B \cot C > 0$, $\cot A$, $\cot B$, $\cot C$ are positive. So, the triangle is acute angled.

4. $\sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{2+2|\cos 2\theta|} = \sqrt{2(1-\cos 2\theta)}$

$= 2|\sin\theta| = 2\sin\theta$ as $\dfrac{\pi}{2} < \theta < \dfrac{3\pi}{4}$

5. $\sec 2\theta = \dfrac{1+\tan^2\theta}{1-\tan^2\theta} = \dfrac{1+n}{1-n}$

where n is a non-square natural number so $1-n \neq 0$. Hence $\sec 2\theta$ is a rational number.

6. $\cos x$ lies between -1 to 1 for all real x.
If $f(x) = \cos(\cos x)$, then $f'(x) = 0$ when either $\sin x = 0$ or $\sin\cos x = 0$, that is, at $x = 0$ or $x = \pi/2$.
At $x = 0$ we get minimum value of $f(x) = \cos 1$

7. $3\tan x = 4\tan y \Rightarrow 3\sin x\cos y = 4\cos x\sin y$
$\Rightarrow 3/4 = 4\cos x\sin y \Rightarrow \cos x\sin y = 3/16$
Therefore,

$$\sin(x+y) = \sin x\cos y + \cos x\sin y = \frac{1}{4} + \frac{3}{16} = \frac{7}{16}$$

8. Maximum value of $4\sin^2 x + 3\cos^2 x$, that is, $\sin^2 x + 3$ is 4 and that of $\sin\dfrac{x}{2} + \cos\dfrac{x}{2}$ is $\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \sqrt{2}$, both attained at $x = \pi/2$.
Hence, the given function has the maximum value $4+\sqrt{2}$.

9. According to the given condition, $\sin\alpha + \sin\beta = -a$ and $\cos\alpha + \cos\beta = -c$. So

$$2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -a \text{ and } 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -c$$

$$\Rightarrow \tan\frac{\alpha+\beta}{2} = \frac{a}{c}$$

Now,

$$\sin(\alpha+\beta) = \frac{2\tan\frac{\alpha+\beta}{2}}{1+\tan^2\frac{\alpha+\beta}{2}} = \frac{2ac}{a^2+c^2}$$

10. $\sin\alpha$, $\sin\beta$, $\cos\alpha$ are in GP. Therefore,
$\sin^2\beta = \sin\alpha\cos\alpha \Rightarrow \cos 2\beta = 1 - \sin 2\beta \geq 0$
Now, the discriminant of the given equation is
$4\cot^2\beta - 4 = 4\cos 2\beta \cdot \text{cosec}^2\beta \geq 0 \Rightarrow$ roots are always real

11. $S = \cos^2\dfrac{\pi}{n} + \cos^2\dfrac{2\pi}{n} + \cdots + \cos^2(n-1)\dfrac{\pi}{n}$

$$= \frac{1}{2}\left[1+\cos\frac{2\pi}{n} + 1+\cos\frac{4\pi}{n} + 1+\cos\frac{6\pi}{n} + \cdots + 1+\cos 2(n-1)\frac{\pi}{n}\right]$$

$$= \frac{1}{2}\left[n-1+\sum_{k=1}^{n-1}\cos\frac{2k\pi}{n}\right] = \frac{1}{2}[n-1-1] = \frac{1}{2}(n-2)$$

12. $\sin A\sin B = \dfrac{1}{2}\times 2\sin A\sin B$

$$= \frac{1}{2}[\cos(A-B) - \cos(A+B)] = \frac{1}{2}[\cos(A-B) - \cos 90°]$$

$$= \frac{1}{2}\cos(A-B) \leq \frac{1}{2}$$

$\Rightarrow$ Maximum value of $\sin A\sin B = \dfrac{1}{2}$

13. $\sin^2 A + \sin^2 B + \sin^2 C = 2 \Rightarrow 2\cos A\cos B\cos C = 0$
$\Rightarrow$ Either $A = 90°$ or $B = 90°$ or $C = 90°$

14. Maximum value of $2\sin x + 4\cos x = 2\sqrt{5}$
Hence, the maximum value of $2\sin x + 4\cos x + 3$ is $2\sqrt{5}+3$

15. $$\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\dfrac{\cos\alpha-\cos\beta}{1-\cos\alpha\cos\beta}}{1+\dfrac{\cos\alpha-\cos\beta}{1-\cos\alpha\cos\beta}}$$

$$= \frac{1-\cos\alpha\cos\beta-\cos\alpha+\cos\beta}{1-\cos\alpha\cos\beta+\cos\alpha-\cos\beta}$$

$$= \frac{(1-\cos\alpha)+\cos\beta(1-\cos\alpha)}{(1+\cos\alpha)-\cos\beta(1+\cos\alpha)} = \frac{(1-\cos\alpha)(1+\cos\beta)}{(1+\cos\alpha)(1-\cos\beta)}$$

$$= \tan^2\frac{\alpha}{2}\cot^2\frac{\beta}{2}$$

Therefore, $\tan\frac{\theta}{2} = \pm\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$.

16. $\sec 2\theta - \tan 2\theta = \dfrac{1-\sin 2\theta}{\cos 2\theta} = \dfrac{1-\dfrac{2\tan\theta}{1+\tan^2\theta}}{\dfrac{1-\tan^2\theta}{1+\tan^2\theta}} = \dfrac{(1-\tan\theta)^2}{1-\tan^2\theta}$

$$= \frac{1-\tan\theta}{1+\tan\theta} = \tan\left(\frac{\pi}{4}-\theta\right)$$

17. Discriminant of the given equation = $(\cos p)^2 - 4(\cos p - 1)\sin p = \cos^2 p + 4(1-\cos p)\sin p \geq 0$, if $p \in (0, \pi)$
[$\because \cos^2 p \geq 0$, $0 \leq 1-\cos p \leq 2$ and $\sin p > 0$ for all $p \in (0, \pi)$]

18. $\sin\dfrac{\pi}{2^n} + \cos\dfrac{\pi}{2^n} = \sqrt{2}\sin\left(\dfrac{\pi}{4}+\dfrac{\pi}{2^n}\right)$ lies in $[-\sqrt{2}, \sqrt{2}]$.

Therefore, $\dfrac{\sqrt{n}}{2} \in [-\sqrt{2}, \sqrt{2}] \Rightarrow \dfrac{\sqrt{n}}{2} \leq \sqrt{2} \Rightarrow \sqrt{n} \leq 2\sqrt{2} \Rightarrow n \leq 8$

Note that $n = 1$ does not satisfy the given equation and for $n > 1$

$$\frac{\pi}{2} \geq \frac{\pi}{4}+\frac{\pi}{2^n} > \frac{\pi}{4} \Rightarrow \sin\left(\frac{\pi}{4}+\frac{\pi}{2^n}\right) > \sin\frac{\pi}{4}$$

$$\Rightarrow \sqrt{2}\sin\left(\frac{\pi}{4}+\frac{\pi}{2^n}\right) > 1 \Rightarrow \frac{\sqrt{n}}{2} > 1 \Rightarrow n > 4$$

Hence, $4 < n \leq 8$.

19. Given $\dfrac{x}{\cos\theta} = \dfrac{y}{\cos\left(\theta+\dfrac{2\pi}{3}\right)} = \dfrac{z}{\cos\left(\theta-\dfrac{2\pi}{3}\right)} = \lambda$ (say)

$$\Rightarrow x+y+z = \lambda\left[\cos\theta + \cos\left(\theta+\frac{2\pi}{3}\right) + \cos\left(\theta-\frac{2\pi}{3}\right)\right]$$

$$= \lambda\left\{\cos\theta + 2\cos\theta\cos\frac{2\pi}{3}\right\} = 0$$

20. $\tan(A+B) = \tan(180° - C)$

$$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B} = -\tan C$$

$$\Rightarrow \tan A + \tan B + \tan C = \tan A\tan B\tan C$$

$$\Rightarrow \frac{\tan A+\tan B+\tan C}{3} \geq \sqrt[3]{\tan A\tan B\tan C} \quad [\because \text{A.M.} \geq \text{G.M.}]$$

$$\Rightarrow \tan A\tan B\tan C \geq 3\sqrt[3]{\tan A\tan B\tan C}$$

$$\Rightarrow \tan^2 A\tan^2 B\tan^2 C \geq 27 \quad \text{[cubing both sides]}$$

$$\Rightarrow \tan A\tan B\tan C \geq 3\sqrt{3}$$

$$\Rightarrow \tan A + \tan B + \tan C \geq 3\sqrt{3}$$

21. From the second equation, we have

$$\sin 2B = \frac{3}{2}\sin 2A \quad (1)$$

and from the first equality

$$3\sin^2 A = 1 - 2\sin^2 B = \cos 2B \quad (2)$$

Now $\cos(A+2B) = \cos A\cdot\cos 2B - \sin A\cdot\sin 2B$

$$= 3\cos A\cdot\sin^2 A - \frac{3}{2}\cdot\sin A\cdot\sin 2A$$

$$= 3\cos A\cdot\sin^2 A - 3\sin^2 A\cdot\cos A = 0$$

$$\Rightarrow A+2B = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

Given that $0 < A < \dfrac{\pi}{2}$ and $0 < B < \dfrac{\pi}{2} \Rightarrow 0 < A+2B < \pi + \dfrac{\pi}{2}$

Hence, $A+2B = \dfrac{\pi}{2}$.

22. $a\cos^3\theta + 3a\cos\theta\sin^2\theta = x$
$a\sin^3\theta + 3a\cos^2\theta\sin\theta = y$
$x+y = a[\sin^3\theta + \cos^3\theta + 3\sin\theta\cos\theta(\sin\theta+\cos\theta)]$
$= a(\sin\theta+\cos\theta)^3$

$$\left(\frac{x+y}{a}\right)^{1/3} = \sin\theta + \cos\theta \quad (1)$$

$x-y = a[\cos^3\theta - \sin^3\theta + 3\cos\theta\sin^2\theta - 3\cos^2\theta\sin\theta]$
$= a[\cos\theta - \sin\theta]^3$

$$\left(\frac{x-y}{a}\right)^{1/3} = \cos\theta - \sin\theta \quad (2)$$

$$(\sin\theta+\cos\theta)^2 + (\cos\theta-\sin\theta)^2 = \frac{(x+y)^{2/3}+(x-y)^{2/3}}{a^{2/3}}$$

$$2(\sin^2\theta+\cos^2\theta) = \frac{(x+y)^{2/3}+(x-y)^{2/3}}{a^{2/3}}$$

$$(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$$

23. Let $a = \sin 4\theta$. Then

$$\sqrt{1+a} = \cos 2\theta + \sin 2\theta$$

and $\sqrt{1-a} = \cos 2\theta - \sin 2\theta$ $(1+\sqrt{1+a})\tan\alpha = (1+\sqrt{1-a})$

$\Rightarrow (1+\cos 2\theta+\sin 2\theta)\tan\alpha = 1+\cos 2\theta - \sin 2\theta$

$$\Rightarrow \frac{2\cos\theta(\cos\theta+\sin\theta)}{2\cos\theta(\cos\theta-\sin\theta)} = \cot\alpha$$

$$\Rightarrow \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \cot\alpha \Rightarrow \frac{1+\tan\theta}{1-\tan\theta} = \cot\alpha$$

$$\Rightarrow \tan\left(\frac{\pi}{4}+\theta\right) = \tan\left(\frac{\pi}{2}-\alpha\right) \Rightarrow \theta = \left(\frac{\pi}{4}-\alpha\right)$$

So $a = \sin 4\theta = \sin(\pi - 4\alpha) = \sin 4\alpha$

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