ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

CLASS 8

SUBJECT :Algebra & Geometry **STUDY MATERIAL 1**

Marks:15 **Exponents**

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EXPONENTS

KEY FACTS

1. The short cut for writing product of a number by itself several times such as, $a \times a \times a \times a \ldots\ldots \times a$ (n times) = a^n, is known as exponential notation, where a is any real number and n is an integer.

   a^n is read as **"a to the nth power".**

   For example : The expression 3^6 is read as **three to the sixth power**. Here, 3 is called the **base** and 6 is called the **exponent** and 3^6 is the **exponential form.**

2. **Laws of exponents**

   If $a \neq 0$, $b \neq 0$ be any real number and m, n be any rational numbers. Then,

   Law I : $a^m \times a^n = a^{m+n}$

   Law II : $(a^m)^n = a^{mn}$

   Law III : $(ab)^m = a^m \times b^m$

   Law IV : $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, (b \neq 0)$

   Law V : $\frac{a^m}{a^n} = a^{m-n}$

   Law VI : $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

   Law VII : $a^0 = 1$

   Law VIII : $a^{-n} = \frac{1}{a^n}$; $a^n = \frac{1}{a^{-n}}$

Solved Examples

Ex. 1. ***Simplify*: $(0.04)^{-1.5}$**

Sol. $(0.04)^{-1.5} = (0.04)^{-\frac{3}{2}} = \frac{1}{(0.04)^{\frac{3}{2}}} = \frac{1}{(\sqrt{0.04})^3} = \frac{1}{(0.2)^3} = \frac{1}{0.008} = \frac{1000}{8} = 125.$

Ex. 2. ***Simplify :*** $(100)^{\frac{1}{2}} \times (0.001)^{\frac{1}{3}} - (0.0016)^{\frac{1}{4}} \times 3^0 + \left(\frac{5}{4}\right)^{-1}$

Sol. Given exp. $= (10^2)^{\frac{1}{2}} \times \left(\frac{1}{1000}\right)^{\frac{1}{3}} - \left(\frac{16}{10000}\right)^{\frac{1}{4}} \times 3^0 + \left(\frac{5}{4}\right)^{-1} = 10^{2\times\frac{1}{2}} \times \left(\frac{1}{10}\right)^{3\times\frac{1}{3}} - \left(\frac{2}{10}\right)^{4\times\frac{1}{4}} \times 3^0 + \frac{4}{5}$

$= 10 \times \frac{1}{10} - \frac{1}{5} + \frac{4}{5} = 10 \times 0.1 - 0.2 + 0.8 = 1 + 0.6 = 1.6.$

Ex. 3. ***Evaluate :*** $\frac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

Sol. Given exp. $= \frac{6^{\frac{2}{3}} \times 6^{\frac{7}{3}}}{6^{\frac{6}{3}}} = 6^{\frac{2}{3}+\frac{7}{3}-\frac{6}{3}} = 6^{\frac{3}{3}} = 6^1 = 6$.

Ex. 4. ***Given that $10^{0.48} = x$ and $10^{0.70} = y$ and $x^z = y^2$, then find the approximate value of z ?***

Sol. Given, $x^z = y^2 \Rightarrow (10^{0.48})^z = (10^{0.70})^2$

$\Rightarrow 10^{0.48z} = 10^{1.40} \Rightarrow 0.48z = 1.40 \Rightarrow z = \frac{140}{48} = 2.9$ (approx)

Ex. 5. ***If $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3}$, then what is the value of n ?***

Sol. Given, $\left(3^{\frac{1}{2}}\right)^5 \times (3^2)^2 = 3^n \times 3 \times 3^{\frac{1}{2}} \Rightarrow 3^{\frac{5}{2}} \times 3^4 = 3^{n+1+\frac{1}{2}}$

$\Rightarrow 3^{\frac{5}{2}+4} = 3^{n+\frac{3}{2}} \Rightarrow 3^{13/2} = 3^{n+3/2} \Rightarrow n + \frac{3}{2} = \frac{13}{2} \Rightarrow n = \frac{13}{2} - \frac{3}{2} = \frac{10}{2} = 5$.

Ex. 6. ***If $3^{x-y} = 27$ and $3^{x+y} = 243$, then what is the value of x?***

Sol. Given, $3^{x-y} = 27 = 3^3$ and $3^{x+y} = 243 = 3^5$

$\Rightarrow$ $x - y = 3$ ... (i)

$x + y = 5$ ... (ii)

Adding equations (i) and (ii), we get

$2x = 8 \Rightarrow x = 4$

Ex. 7. ***Simplify :*** $\left(\frac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a)}$

Sol. $\left(\frac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\frac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a)} = \left(x^{(a-b)}\right)^{(a+b)} \cdot \left(x^{(b-c)}\right)^{(b+c)} \cdot \left(x^{(c-a)}\right)^{(c+a)}$

$= x^{a^2-b^2} . x^{b^2-c^2} . x^{c^2-a^2} = x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1.$

Ex. 8. ***If $2 = 10^m$ and $3 = 10^n$, then find the value of 0.15.***

Sol. $0.15 = \frac{1.5}{10} = \frac{3}{2 \times 10} = \frac{10^n}{10^m \times 10} = \frac{10^n}{10^{m+1}} = 10^{n-(m+1)} = 10^{n-m-1}$

Ex. 9. ***What is the value of the expression*** $\frac{x-1}{x^{3/4}+x^{1/2}} \cdot \frac{x^{1/2}+x^{1/4}}{x^{1/2}+1} \cdot x^{1/4}$ ***when*** $x = 16$ ***?***

Sol. Required value $= \frac{16-1}{16^{3/4}+16^{1/2}} \cdot \frac{16^{1/2}+16^{1/4}}{16^{1/2}+1} \cdot 16^{1/4}$

$$= \frac{15}{(2^4)^{3/4}+(4^2)^{1/2}} \cdot \frac{(4^2)^{1/2}+(2^4)^{1/4}}{(4^2)^{1/2}+1} \cdot (2^4)^{1/4} = \frac{15}{2^3+4} \cdot \frac{4+2}{4+1} \cdot 2 = \frac{15}{12} \times \frac{6}{5} \times 2 = \mathbf{3}.$$

Ex. 10. ***What is the simplified value of*** $\left\{\frac{4^{m+\frac{1}{4}} \times \sqrt{2 \cdot 2^m}}{2\sqrt{2^{-m}}}\right\}^{\frac{1}{m}}$ ***?***

Sol. $\left\{\frac{4^{m+\frac{1}{4}} \times \sqrt{2 \cdot 2^m}}{2\sqrt{2^{-m}}}\right\}^{\frac{1}{m}} = \left\{\frac{(2^2)^{m+\frac{1}{4}} \times (2^{m+1})^{\frac{1}{2}}}{2 \times 2^{\frac{-m}{2}}}\right\}^{\frac{1}{m}} = \left\{\frac{2^{2m+\frac{1}{2}} \times 2^{\frac{m}{2}+\frac{1}{2}}}{2^{1-\frac{m}{2}}}\right\}^{\frac{1}{m}} = \left\{2^{2m+\frac{1}{2}+\frac{m}{2}+\frac{1}{2}-1+\frac{m}{2}}\right\}^{\frac{1}{m}} = (2^{3m})^{\frac{1}{m}} = 2^3 = \mathbf{8}.$

Ex. 11. ***Given,*** $a = 2^x$, $b = 4^y$, $c = 8^z$ ***and*** $ac = b^2$. ***Find the relation between*** x, y ***and*** z

Sol. $ac = b^2$

$\Rightarrow 2^x.8^z = (4^y)^2 \Rightarrow 2^x.(2^3)^z = ((2^2)^y)^2 \Rightarrow 2^x.2^{3z} = 2^{4y}$

$\Rightarrow 2^{x+3z} = 2^{4y} \Rightarrow x + 3z = 4y.$

QUESTION BANK

1. $\frac{\left(\frac{1}{216}\right)^{\frac{-2}{3}}}{\left(\frac{1}{27}\right)^{\frac{-4}{3}}} = x$. The value of x is

(a) $\frac{3}{4}$ (b) $\frac{4}{9}$

(c) $\frac{2}{3}$ (d) $\frac{1}{8}$

2. If $\sqrt{3^n} = 81$. Then, n is equal to

(a) 2 (b) 4

(c) 6 (d) 8

3. $(64)^{\frac{-2}{3}} \times \left(\frac{1}{4}\right)^{-3}$ equals

(a) $\frac{1}{4}$ (b) 1

(c) 4 (d) 16

4. $\frac{1}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{1}{(243)^{\frac{-1}{5}}}$ is equal to

(a) 103 (b) 105

(c) 107 (d) 101

5. $(4)^{0.5} \times (0.5)^4$ is equal to

(a) 1 (b) 4

(c) $\frac{1}{8}$ (d) $\frac{1}{32}$

6. $\left(\frac{1}{64}\right)^0 + (64)^{\frac{-1}{2}} + (32)^{\frac{4}{5}} - (32)^{\frac{-4}{5}}$ is equal to

(a) $16\frac{1}{8}$ (b) $17\frac{1}{8}$

(c) $17\frac{1}{16}$ (d) $-17\frac{1}{16}$

7. Simplify :

$$\left[\left(2\frac{10}{27}\right)^{-\frac{2}{3}} \div \left(11\frac{1}{9}\right)^{-0.5}\right] + \left[(6.25)^{0.5} \div (-4)^{-1}\right]$$

(a) $-8\frac{1}{8}$ (b) $8\frac{1}{8}$

(c) $1\frac{7}{8}$ (d) $-1\frac{7}{8}$

8. **Simplify :** $\dfrac{(6.25)^{\frac{1}{2}} \times (0.0144)^{\frac{1}{2}} + 1}{(0.027)^{\frac{1}{3}} \times (81)^{\frac{1}{4}}}$

(a) 0.14 (b) 1.4

(c) 1 (d) $1.\overline{4}$

9. $4^{3.5} : 2^5$ is the same as

(a) 4 : 1 (b) 2 : 1

(c) 7 : 5 (d) 7 : 10

10. **Simplify :** $\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{5^9}}\right]^4$

(a) 5^2 (b) 5^4

(c) 5^8 (d) 5^{12}

11. The value of $\dfrac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \cdot 3^{n-1}}$ is

(a) 1 (b) 9

(c) 3 (d) 3^n

12. If $x^{x\sqrt{x}} = (x\sqrt{x})^x$. then x is equal to

(a) $\dfrac{3}{2}$ (b) $\dfrac{2}{9}$

(c) $\dfrac{9}{4}$ (d) $\dfrac{4}{9}$

13. $[1-\{1-(1-a^4)^{-1}\}^{-1}]^{-\frac{1}{4}}$ is equal to

(a) a^4 (b) a^2

(c) a (d) $\dfrac{1}{a}$

14. If $64^a = \dfrac{1}{256^b}$, then $3a + 4b$ equals

(a) 2 (b) 4

(c) 8 (d) 0

15. If $a = b^{\frac{2}{3}}$ and $b = c^{-2}$, what is the value of a in terms of c ?

(a) $\dfrac{4}{c^3}$ (b) $\sqrt[3]{c^4}$

(c) $\dfrac{1}{\sqrt[3]{c^4}}$ (d) $\sqrt[4]{c^3}$

16. The value of $\dfrac{5\cdot(25)^{n+1} + 25\cdot(5)^{2n-1}}{25\cdot(5)^{2n} - 105(25)^{n-1}}$ is

(a) 0 (b) 1

(c) $6\dfrac{1}{4}$ (d) $5\dfrac{1}{4}$

17. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is

(a) 4 (b) 5

(c) 6 (d) 8

18. What is the expression $(x + y)^{-1}\ (x^{-1}+y^{-1})\ (xy^{-1} + x^{-1}y)^{-1}$ equal to

(a) $x + y$ (b) $(x^2 + y^2)^{-1}$

(c) xy (d) $x^2 + y^2$

19. If $2^x - 2^{x-1} = 4$, then what is the value of $2^x + 2^{x-1}$?

(a) 8 (b) 12

(c) 10 (d) 16

20. If $x = y^z$, $y = z^x$ and $z = x^y$, then

(a) $\dfrac{xy}{z} = 1$ (b) $xyz = 1$

(c) $x + y + z = 1$ (d) $xz = y$

21. $\dfrac{1}{1+x^{b-a}+x^{c-a}} + \dfrac{1}{1+x^{a-b}+x^{c-b}} + \dfrac{1}{1+x^{b-c}+x^{a-c}}$ equals.

(a) $\dfrac{1}{2}$ (b) 2

(c) 1 (d) 0

22. Find the value of x if $\left[3^{2x-2} + 10\right] \div 13 = 7$.

(a) 1 (b) 3

(c) 4 (d) 2

23. The value of $\dfrac{2^{\frac{1}{2}} \cdot 3^{\frac{1}{3}} \cdot 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \cdot 5^{\frac{3}{5}}} \div \dfrac{3^{\frac{4}{3}} \cdot 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \cdot 6}$ is

(a) 5 (b) 6

(c) 10 (d) 15

24. The value of $\left(x^{\frac{b+c}{c-a}}\right)^{\frac{1}{a-b}} \left(x^{\frac{c+a}{a-b}}\right)^{\frac{1}{b-c}} \left(x^{\frac{a+b}{b-c}}\right)^{\frac{1}{c-a}}$ is

(a) 1 (b) a

(c) b (d) c

25. The value of $\left(\dfrac{x^q}{x^r}\right)^{\frac{1}{qr}} \times \left(\dfrac{x^r}{x^p}\right)^{\frac{1}{rp}} \times \left(\dfrac{x^p}{x^q}\right)^{\frac{1}{pq}}$ is equal to

(a) $x^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$ (b) 0

(c) $x^{pq+qr+rp}$ (d) 1

26. The largest number among the following is

(a) $3^{2^{2^2}}$ (b) $\left\{(3^2)^2\right\}^2$

(c) $3^2 \times 3^2 \times 3^2$ (d) 3222

27. If $6^x - 6^{x-3} = 7740$, then $x^x =$

(a) 7796 (b) 243

(c) 3125 (d) 46656

28. The value of

$$\frac{9^x(9^{x-1})^x}{9^{x+1}\cdot 3^{2x-2}}\left\{\frac{729^{\frac{x}{3}}}{81}\right\}^{-x} \div \frac{3^a - 2^3\cdot 3^{a-2}}{3^a - 3^{a-1}}$$

(a) 9 (b) 6

(c) 12 (d) 16

29. Find the value of $(2^{\frac{1}{4}} - 1)(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1)$

(a) 2 (b) 3

(c) 5 (d) 1

30. Simplify : $\frac{a^{\frac{1}{2}} + a^{-\frac{1}{2}}}{1-a} + \frac{1-a^{-\frac{1}{2}}}{1-\sqrt{a}}$

(a) 1 (b) 0

(c) $\frac{2}{1-a}$ (d) $1 + a$

31. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then the value of x and y are

(a) $\frac{17}{8}, \frac{9}{8}$ (b) $\frac{17}{8}, \frac{15}{8}$

(c) $\frac{17}{8}, \frac{11}{8}$ (d) $\frac{15}{8}, \frac{11}{8}$

32. Find x, if $8^{x-2} \times \left(\frac{1}{2}\right)^{4-3x} = (0.0625)^x$

(a) 0 (b) 4

(c) 2 (d) 1

33. Find the value of the expression :

$$\frac{(x^{a+b})^2 \times (x^{b+c})^2 \times (x^{c+a})^2}{(x^a x^b x^c)^4}$$

if $x = 2, a = 1, b = 2, c = 3$

(a) 16 (b) 32

(c) 24 (d) 1

34. If $(2.4)^x = (0.24)^y = 10^z$ then show that $\frac{1}{x} - \frac{1}{z} = \frac{1}{y}$

35. If $2^x = 4^y = 8^z$ and $xyz = 288$, then $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z}$ equals

(a) $\frac{11}{8}$ (b) $\frac{11}{24}$

(c) $\frac{11}{48}$ (d) $\frac{11}{96}$

Answers

1. (b)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (b)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (b)	**19.** (b)	**20.** (b)
21. (c)	**22.** (b)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (a)	**27.** (c)	**28.** (b)	**29.** (d)	**30.** (c)
31. (b)	**32.** (d)	**33.** (d)	**35.** (d)						

Hints and Solutions

1. (b) $\frac{\left(\frac{1}{216}\right)^{-\frac{2}{3}}}{\left(\frac{1}{27}\right)^{-\frac{4}{3}}} = \frac{\left(\left(\frac{1}{6}\right)^3\right)^{-\frac{2}{3}}}{\left(\left(\frac{1}{3}\right)^3\right)^{-\frac{4}{3}}} = \frac{\frac{1}{6^{-2}}}{\frac{1}{3^{-4}}} = \frac{6^2}{3^4} = \frac{36}{81} = \frac{4}{9}.$

2. (d) $\sqrt{3^n} = 81 \Rightarrow 3^{n/2} = 3^4 \Rightarrow \frac{n}{2} = 4 \Rightarrow n = 8$

3. (c) $(64)^{-\frac{2}{3}} \times \left(\frac{1}{4}\right)^{-3} = (4^3)^{-\frac{2}{3}} \times (4^{-1})^{-3}$

$= 4^{-2} \times 4^3 = 4^{-2+3} = 4^1 = 4$

4. (a) Given exp. $= \frac{1}{(6^3)^{-\frac{2}{3}}} + \frac{1}{(4^4)^{-\frac{3}{4}}} + \frac{1}{(3^5)^{-\frac{1}{5}}}$

$= \frac{1}{6^{-2}} + \frac{1}{4^{-3}} + \frac{1}{3^{-1}}$

$= 6^2 + 4^3 + 3 = 36 + 64 + 3 = 103$

5. (c) $(4)^{0.5} \times (0.5)^4 = (2^2)^{0.5} \times \left(\frac{1}{2}\right)^4$

$= 2^1 \times 2^{-4} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}.$

6. (c) Given exp. $= 1 + (8^2)^{-\frac{1}{2}} + (2^5)^{\frac{4}{5}} - (2^5)^{-\frac{4}{5}}$

$= 1 + 8^{-1} + 2^4 - 2^{-4}$

$= 1 + \frac{1}{8} + 16 - \frac{1}{16} = \frac{16 + 2 + 256 - 1}{16}$

$= \frac{273}{16} = 17\frac{1}{16}$

7. (a) Given exp.

$= \left[\left\{\left(\frac{64}{27}\right)^{-\frac{2}{3}} \div \left(\frac{100}{9}\right)^{-\frac{1}{2}}\right\}\right] + \left[(6.25)^{\frac{1}{2}} \div \frac{1}{-4}\right]$

$= \left[\left\{\left(\left(\frac{4}{3}\right)^3\right)^{-\frac{2}{3}} \div \left(\left(\frac{10}{3}\right)^2\right)^{-\frac{1}{2}}\right\}\right] + \left[\left((2.5)^2\right)^{\frac{1}{2}} \div \frac{1}{-4}\right]$

$= \left[\left(\frac{4}{3}\right)^{-2} \div \left(\frac{10}{3}\right)^{-1}\right] + 2.5 \times -4$

$= \left[\left(\frac{3}{4}\right)^2 \div \frac{3}{10}\right] - 10 = \frac{9}{16} \times \frac{10}{3} - 10$

$= \frac{15}{8} - 10 = \frac{15 - 80}{8} = \frac{-65}{8} = -8\frac{1}{8}.$

8. (d) Given exp. $= \frac{\left((2.5)^2\right)^{\frac{1}{2}} \times \left((0.12)^2\right)^{\frac{1}{2}} + 1}{\left((0.3)^3\right)^{\frac{1}{3}} \times (3^4)^{\frac{1}{4}}}$

$= \frac{2.5 \times 0.12 + 1}{0.3 \times 3}$

$= \frac{0.3 + 1}{0.9} = \frac{1.3}{0.9} = \frac{13}{9} = 1.\overline{4}.$

9. (a) $4^{3.5} : 2^5 = (2^2)^{3.5} : 2^5 = 2^7 : 2^5$

$= \frac{2^7}{2^5} : 1 = 2^{7-5} : 1 = 2^2 : 1 = 4 : 1.$

10. (b) $\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4 \times \left[\sqrt[6]{\sqrt[3]{5^9}}\right]^4$

$= 5^{9 \times \frac{1}{6} \times \frac{1}{3} \times 4} \times 5^{9 \times \frac{1}{3} \times \frac{1}{6} \times 4}$

$= 5^2 \times 5^2 = 5^4.$

11. (b) Given exp. $= \frac{(3^5)^{\frac{n}{5}} \cdot 3^{2n+1}}{(3^2)^n \cdot 3^{n-1}} = \frac{3^n \cdot 3^{2n+1}}{3^{2n} \cdot 3^{n-1}}$

$= \frac{3^{3n+1}}{3^{3n-1}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9.$

12. (c) $x^{x\sqrt{x}} = (x\sqrt{x})^x$

$\Rightarrow x^{x\sqrt{x}} = (x^{3/2})^x \Rightarrow x^{x\sqrt{x}} = x^{3/2x}$

$\Rightarrow x\sqrt{x} = \frac{3}{2}x \Rightarrow \sqrt{x} = \frac{3}{2} \Rightarrow x = \frac{9}{4}.$

13. (c) Given exp. $= \left[1 - \left\{1 - \frac{1}{1 - a^4}\right\}^{-1}\right]^{-\frac{1}{4}}$

$= \left[1 - \left\{\frac{1 - a^4 - 1}{1 - a^4}\right\}^{-1}\right]^{-\frac{1}{4}}$

$= \left[1 - \left\{\frac{-a^4}{1 - a^4}\right\}^{-1}\right]^{-\frac{1}{4}} = \left[1 - \left\{\frac{1 - a^4}{-a^4}\right\}\right]^{-\frac{1}{4}}$

$= \left[1 - \left\{\frac{a^4 - 1}{a^4}\right\}\right]^{-\frac{1}{4}} = \left[\frac{a^4 - a^4 + 1}{a^4}\right]^{-\frac{1}{4}}$

$= \left(\frac{1}{a^4}\right)^{-\frac{1}{4}} = (a^{-4})^{-\frac{1}{4}} = a^1 = a.$

14. (d) $64^a = \frac{1}{256^b} \Rightarrow (2^6)^a = \frac{1}{(2^8)^b}$

$\Rightarrow 2^{6a} \times 2^{8b} = 1 \Rightarrow 2^{6a + 8b} = 2^0$

$\Rightarrow 6a + 8b = 0 \Rightarrow 3a + 4b = 0$

15. (c) $a = b^{\frac{2}{3}}$ and $b = c^{-2}$

$\therefore a = (c^{-2})^{\frac{2}{3}} = c^{-\frac{4}{3}} = \frac{1}{c^{4/3}} = \frac{1}{\sqrt[3]{c^4}}$

16. (c) Given exp. $= \dfrac{5\times(5^2)^{n+1}+5^2\times5^{2n-1}}{5^2\times5^{2n}-21\times5\times(5^2)^{n-1}}$

$= \dfrac{5^{2n+3}+5^{2n+1}}{5^{2n+2}-21\times5^{2n-1}} = \dfrac{5^{2n+1}(5^2+1)}{5^{2n-1}(5^3-21)}$

$= \dfrac{5^{(2n+1)-(2n-1)}\times26}{(125-21)} = \dfrac{5^2\times26}{104} = \dfrac{25}{4} = 6\dfrac{1}{4}.$

17. (a) $5\sqrt{5}\times5^3\div5^{-\frac{3}{2}} = 5^{a+2}$

$\Rightarrow 5^{\frac{3}{2}}\times5^3\div5^{-\frac{3}{2}} = 5^{a+2}$

$\Rightarrow 5^{\frac{3}{2}+3-\left(-\frac{3}{2}\right)} = 5^{a+2}$

$\Rightarrow 5^6 = 5^{a+2} \Rightarrow a+2=6 \Rightarrow a=4$

18. (b) $(x+y)^{-1}(x^{-1}+y^{-1})(xy^{-1}+x^{-1}y)^{-1}$

$= \dfrac{1}{(x+y)}\times\left(\dfrac{1}{x}+\dfrac{1}{y}\right)\times\left(\dfrac{x}{y}+\dfrac{y}{x}\right)^{-1}$

$= \dfrac{1}{(x+y)}\times\left(\dfrac{y+x}{xy}\right)\times\left(\dfrac{x^2+y^2}{xy}\right)$

$= \dfrac{1}{\cancel{(x+y)}}\times\dfrac{\cancel{(y+x)}}{\cancel{xy}}\times\dfrac{\cancel{xy}}{(x^2+y^2)}$

$= (x^2+y^2)^{-1}$

19. (b) $2^x - 2^{x-1} = 4 \Rightarrow 2^x - \dfrac{2^x}{2} = 4$

$\Rightarrow 2^x\left(1-\dfrac{1}{2}\right)=4 \Rightarrow 2^x\times\dfrac{1}{2}=4$

$\Rightarrow 2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$

$\therefore\ 2^x + 2^{x-1} = 2^3 + 2^2 = 8+4 = 12.$

20. (b) $z = x^y = (y^z)^y$ $(\because x = y^z)$

$= y^{zy} = (z^x)^{zy} = z^{xyz}$ $(\because y = z^x)$

$\therefore\ z^1 = z^{xyz} \Rightarrow xyz = 1$

21. (c) $\dfrac{1}{1+x^{b-a}+x^{c-a}} + \dfrac{1}{1+x^{a-b}+x^{c-b}} + \dfrac{1}{1+x^{b-c}+x^{a-c}}$

$= \dfrac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \dfrac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \dfrac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$

$= \dfrac{x^a}{x^a+x^b+x^c} + \dfrac{x^b}{x^b+x^a+x^c} + \dfrac{x^c}{x^c+x^b+x^a}$

$= \dfrac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1.$

22. (b) $\left[3^{2x-2}+10\right]\div13 = 7$

$\Rightarrow 3^{2x-2}+10 = 7\times13 = 91$

$\Rightarrow 3^{2x-2} = 91-10 = 81 = 3^4$

$\Rightarrow 2x-2 = 4 \Rightarrow 2x = 6 \Rightarrow x = 3.$

23. (c) Given exp. $= \dfrac{2^{\frac{1}{2}}\times3^{\frac{1}{3}}\times(2^2)^{\frac{1}{4}}}{(2\times5)^{-\frac{1}{5}}\times5^{\frac{3}{5}}} \div \dfrac{3^{\frac{4}{3}}\times5^{-\frac{7}{5}}}{(2^2)^{\frac{-3}{5}}\times2\times3}$

$= \dfrac{2^{\frac{1}{2}}\times3^{\frac{1}{3}}\times(2)^{\frac{1}{2}}}{(2)^{-\frac{1}{5}}\times5^{-\frac{1}{5}}\times5^{\frac{3}{5}}} \times \dfrac{(2)^{\frac{-6}{5}}\times2\times3}{3^{\frac{4}{3}}\times5^{-\frac{7}{5}}}$

$= \dfrac{2^{\left(\frac{1}{2}+\frac{1}{2}-\frac{6}{5}+1+\frac{1}{5}\right)}\times3^{\left(\frac{1}{3}+1-\frac{4}{3}\right)}}{5^{\left(-\frac{1}{5}+\frac{3}{5}-\frac{7}{5}\right)}}$

$= \dfrac{2\times3^0}{5^{-1}} = 2\times5 = 10.$

24. (a) $\left(x^{\frac{b+c}{c-a}}\right)^{\frac{1}{a-b}}\times\left(x^{\frac{c+a}{a-b}}\right)^{\frac{1}{b-c}}\times\left(x^{\frac{a+b}{b-c}}\right)^{\frac{1}{c-a}}$

$= x^{\frac{b+c}{(c-a)(a-b)}}\times x^{\frac{c+a}{(a-b)(b-c)}}\times x^{\frac{a+b}{(b-c)(c-a)}}$

$= x^{\frac{b+c}{(c-a)(a-b)}+\frac{c+a}{(a-b)(b-c)}+\frac{a+b}{(b-c)(c-a)}}$

$= x^{\frac{(b+c)(b-c)+(c+a)(c-a)+(a+b)(a-b)}{(a-b)(b-c)(c-a)}}$

$= x^{\frac{b^2-c^2+c^2-a^2+a^2-b^2}{(a-b)(b-c)(c-a)}}$

$= x^{\frac{0}{(a-b)(b-c)(c-a)}} = x^0 = 1.$

25. (d) $\left(\dfrac{x^q}{x^r}\right)^{\frac{1}{qr}}\times\left(\dfrac{x^r}{x^p}\right)^{\frac{1}{rp}}\times\left(\dfrac{x^p}{x^q}\right)^{\frac{1}{pq}}$

$= (x^{q-r})^{\frac{1}{qr}}\times(x^{r-p})^{\frac{1}{rp}}\times(x^{p-q})^{\frac{1}{pq}}$

$= x^{\frac{q-r}{qr}}\times x^{\frac{r-p}{rp}}\times x^{\frac{p-q}{pq}}$

$= x^{\frac{1}{r}-\frac{1}{q}}\times x^{\frac{1}{p}-\frac{1}{r}}\times x^{\frac{1}{q}-\frac{1}{p}}$

$= x^{\frac{1}{r}-\frac{1}{q}+\frac{1}{p}-\frac{1}{r}+\frac{1}{q}-\frac{1}{p}} = x^0 = 1.$

26. (a) $3^{2^{2^2}} = 3^{2^4} = 3^{16}$;

$\left\{\left(3^2\right)^2\right\}^2 = 3^8 = 6561$;

$3^2 \times 3^2 \times 3^2 = 3^{2+2+2} = 3^6 = 729$;

$\therefore\ 3^{16} > 3^8 > 3222 > 3^6$

27. (c) $6^x - 6^{x-3} = 7740 \Rightarrow\ 6^x - \dfrac{6^x}{6^3} = 7740$

$\Rightarrow 6^x\left(1 - \dfrac{1}{216}\right) = 7740 \Rightarrow 6^x \times \dfrac{215}{216} = 7740$

$\Rightarrow 6^x = \dfrac{7740 \times 216}{215} = 36 \times 216 = 6^5 \Rightarrow x = 5$

$\therefore\ x^x = 5^5 = 3125.$

28. (b) Given exp.

$$= \frac{3^{2x}(3^{2x-2})^x}{3^{2x+2} \cdot 3^{2x-2}}\left\{\frac{(3^6)^{\frac{x}{3}}}{3^4}\right\}^{-x} \div \frac{3^{a-2}(3^2 - 2^3)}{3^{a-1}(3-1)}$$

$$= \frac{3^{2x} \cdot 3^{2x^2-2x}}{3^{2x+2+2x-2}}\left\{\frac{3^{2x}}{3^4}\right\}^{-x} \div \frac{3^{a-2-a+1} \times (9-8)}{2}$$

$$= \frac{3^{2x^2}}{3^{4x}}(3^{2x-4})^{-x} \div \frac{1}{2 \times 3}$$

$$= 3^{2x^2-4x} \times 3^{-2x^2+4x} \div \frac{1}{6} = 3^0 \times 6 = 1 \times 6 = 6.$$

29. (d) $\left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right)$

Let $2^{\frac{1}{4}} = a$. Then,

Given exp. $= (a-1)(a^3 + a^2 + a + 1)$

$= (a-1)(a^2(a+1) + 1(a+1))$

$= (a-1)(a+1)(a^2+1) = (a^2-1)(a^2+1)$

$= a^4 - 1$

$\therefore$ Required value $= \left(2^{\frac{1}{4}}\right)^4 - 1 = 2 - 1 = 1.$

30. (c) $\dfrac{a^{\frac{1}{2}} + a^{\frac{-1}{2}}}{1-a} + \dfrac{1 - a^{\frac{-1}{2}}}{1+\sqrt{a}} = \dfrac{\sqrt{a} + \frac{1}{\sqrt{a}}}{1-a} + \dfrac{1 - \frac{1}{\sqrt{a}}}{1+\sqrt{a}}$

$$= \frac{\frac{a+1}{\sqrt{a}}}{1-a} + \frac{\frac{\sqrt{a}-1}{\sqrt{a}}}{1+\sqrt{a}} = \frac{a+1}{\sqrt{a}(1-a)} + \frac{\sqrt{a}-1}{\sqrt{a}(1+\sqrt{a})}$$

$$= \frac{(a+1)(1+\sqrt{a}) + (\sqrt{a}-1)(1-a)}{\sqrt{a}(1-a)(1+\sqrt{a})}$$

$$= \frac{(a+1+a\sqrt{a}+\sqrt{a}) + \sqrt{a} - 1 - a\sqrt{a} + a}{\sqrt{a}(1-a)(1+\sqrt{a})}$$

$$= \frac{2a + 2\sqrt{a}}{\sqrt{a}(1-a)(1+\sqrt{a})}$$

$$= \frac{2\sqrt{a} + (\sqrt{a}+1)}{\sqrt{a}(1-a)(1+\sqrt{a})} = \frac{2}{1-a}.$$

31. (b) $3^{x+y} = 81 \Rightarrow 3^{x+y} = 3^4 = x + y = 4$ ... (*i*)

$81^{x-y} = 3 \Rightarrow (3^4)^{x-y} = 3^1$

$\Rightarrow 4x - 4y = 1$ ... (*ii*)

Eqn (*i*) × 4 + Eqn (*ii*) gives

$4x + 4y + 4x - 4y = 16 + 1$

$\Rightarrow 8x = 17 \Rightarrow x = \dfrac{17}{8}$

Putting $x = \dfrac{17}{8}$ in (*i*), we get $\dfrac{17}{8} + y = 4$

$\Rightarrow\ y = 4 - \dfrac{17}{8} = \dfrac{15}{8}$

$\therefore\ x = \dfrac{17}{8}, y = \dfrac{15}{8}.$

32. (d) $8^{x-2} \times \left(\dfrac{1}{2}\right)^{4-3x} = (0.0625)^x$

$\Rightarrow (2^3)^{x-2} \times (2^{-1})^{4-3x} = \left(\dfrac{625}{10000}\right)^x$

$\Rightarrow\ 2^{3x-6} \times 2^{-4+3x} = \left(\dfrac{1}{16}\right)^x = (2^{-4})^x = 2^{-4x}$

$\Rightarrow 2^{3x-6-4+3x} = 2^{-4x}$

$\Rightarrow 2^{6x-10} = 2^{-4x}$

$\Rightarrow 6x - 10 = -4x$

$\Rightarrow 10x = 10 \Rightarrow x = 1.$

33. (d) $\dfrac{(x^{a+b})^2 \times (x^{b+c})^2 \times (x^{c+a})^2}{(x^a\, x^b\, x^c)^4}$

$$= \frac{x^{2a+2b} \times x^{2b+2c} \times x^{2c+2a}}{(x^{a+b+c})^4}$$

$$= \frac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}} = \frac{x^{4a+4b+4c}}{x^{4a+4b+4c}} = 1.$$

34. $(2.4)^x = 10^z \Rightarrow 2.4 = 10^{\frac{z}{x}}$

and $(0.24)^y = 10^z \Rightarrow 0.24 = 10^{\frac{z}{y}}$

$\therefore \quad \frac{2.4}{0.24} = \frac{10^{\frac{z}{x}}}{10^{\frac{z}{y}}} \Rightarrow 10 = 10^{\frac{z}{x}-\frac{z}{y}}$

$\Rightarrow 1 = \frac{z}{x} - \frac{z}{y} = z\left(\frac{1}{x} - \frac{1}{y}\right).$

$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{z}$ or $\frac{1}{x} - \frac{1}{z} = \frac{1}{y}.$

35. (d) $2^x = (2^2)^y = (2^3)^z \Rightarrow x = 2y = 3z$

Given $xyz = 288 \Rightarrow x \times \frac{x}{2} \times \frac{x}{3} = 288$

$\Rightarrow x^3 = 6 \times 288 \Rightarrow x^3 = 1728$

$\Rightarrow x = \sqrt[3]{1728} = 12$

$\therefore \quad y = \frac{12}{2} = 6$ and $z = \frac{12}{3} = 4$

$\therefore \quad \frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = \frac{1}{24} + \frac{1}{24} + \frac{1}{32}$

$= \frac{4+4+3}{96} = \frac{11}{96}.$

SELF ASSESSMENT EXERCISE

1. Find the value of $(27)^{-2/3} + ((2^{-2/3})^{-5/3})^{-9/10}$

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{11}{18}$ (d) 1

2. Given that: $10^{0.48} = x$ and $10^{0.7} = y$ and $x^z = y^2$, find the value of z.

(a) $2\frac{11}{12}$ (b) $\frac{4}{9}$

(c) $1\frac{1}{48}$ (d) $\frac{48}{49}$

3. The expression $(x^{-2p}\ y^{3q})^6 \div (x^3y^{-1})^{-4p}$ after simplification becomes:

(a) independent of x but not of y.

(b) independent of y but not of x.

(c) independent of both x and y.

(d) dependent of both x and y but independent of p and q.

4. If $2^{x-1} + 2^{x+1} = 320$, then x equals

(a) 4 (b) 5

(c) 6 (d) 7

5. $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$ equals

(a) $a^3 + b^3 + c^3$ (b) 1

(c) $(a + b + c)^3$ (d) 0

6. $\frac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$ equals

(a) $\frac{1}{4}$ (b) $-\frac{1}{2}$

(c) $-\frac{1}{4}$ (d) $\frac{1}{2}$

7. If $3^{x+8} = 27^{2x+1}$, then the value of

$\left[\left(\frac{\sqrt{289}}{\sqrt[3]{216}}\right)^x \div \left(\frac{17}{\sqrt[4]{1296}}\right)^x\right]^{1/2}$ is

(a) 1 (b) 0

(c) $\frac{17}{6}$ (d) $\frac{6}{17}$

8. Find the value of

$$\frac{(0.3)^{1/3}\left(\frac{1}{27}\right)^{1/4}(9)^{1/6}(0.81)^{2/3}}{(0.9)^{2/3}(3)^{-1/2}(243)^{-1/4}}$$

(a) 0.9 (b) 2.7

(c) 0.27 (d) 0.09

9. $\frac{\sqrt[6]{2}\left[(625)^{3/5} \times (1024)^{-6/5} \div (25)^{3/5}\right]^{1/2}}{[(\sqrt[3]{128})^{-5/2}] \times (125)^{1/5}} \div \frac{(10^3)^2 \div (10^2)^3}{(10^2)^3 \div (10^3)^2}$ equals

(a) $\frac{1}{5}$ (b) $\frac{1}{125}$

(c) 1 (d) $\frac{1}{10}$

10. If $a = (\sqrt{5} + \sqrt{4})^{-3}$ and $b = (\sqrt{5} - \sqrt{4})^{-3}$, then the value of $(a + 1)^{-1} + (b + 1)^{-1}$ is

(a) $20\sqrt{5}$ (b) 4

(c) 1 (d) $16\sqrt{5}$

Answers

1. (c)	2. (a)	3. (a)	4. (d)	5. (b)	6. (d)	7. (a)	8. (b)	9. (c)	10. (c)