ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

CLASS 8

SUBJECT :Arithmetic **STUDY MATERIAL 1**

Marks:15 **Squares and Cube roots**

Date:4.5.2020

SQUARES & SQUARE ROOTS, CUBES & CUBE ROOTS

KEY FACTS

1. If a number is multiplied by itself, the product so obtained is called the square of that number. It is a number raised to the power 2.

   In the statement 13 × 13 = 169, 169 is the **square** of 13 and 13 is the **square root** of 169.

2. The square of a natural number is called a **perfect square**. Following are some important properties of square numbers.

   (*i*) A square number is never negative.

   (*ii*) A square number never ends in 2, 3, 7 or 8.

   (*iii*) The number of zeros at the end of a perfect square is always even.

   (*iv*) The square of an even number is even.

   (*v*) The square of an odd number is odd.

   (*vi*) For any natural number n. n^2 = Sum of first n odd natural numbers.

3. **Finding the square root :**

   (*i*) The square root of a perfect square number can be obtained by finding the prime factorization of the square number, pairing equal factors and picking out one prime factor of each pair. The product of the prime factors thus picked gives the square root of the number.

> **Note:** We may also write the product of prime factors in exponential form and for finding the square root, we take half of the index value of each factor and then multiply.

For example :

$196 = \underline{2 \times 2} \times \underline{7 \times 7} \Rightarrow \sqrt{196} = 2 \times 7 = 14$ or

$196 = 2^2 \times 7^2 \Rightarrow \sqrt{196} = 2^{2/2} \times 7^{2/2} = 2 \times 7 = 14$

(*ii*) $\sqrt{p \times q} = \sqrt{p} \times \sqrt{q}$ (*iii*) $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$

(*iv*) The square root of a number can also be found by ***division method.*** You will be explained this method with the help of an example.

Example: Find the square root of 17424.

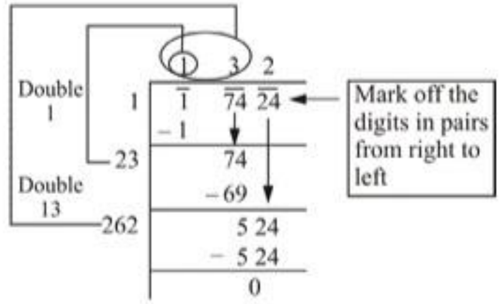
Step 1. Take the first pair of digits and find the nearest perfect square. Here $1^2 = 1$.

Step 2. Twice of 1 = 2

Step 3. 2 goes into 7 three times. Put 3 on the top and in the divisor as shown. $23 \times 3 = 69$.

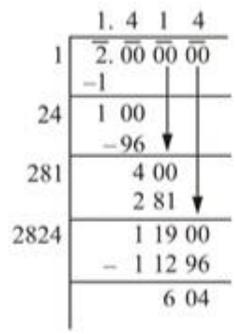
Step 4. Double 13. You get 26. 26 goes into 52, 2 times. Place 2 on top and in the divisor as shown $2 \times 262 = 524$.

Step 5. Subtract. The remainder is 0. Therefore, 132 is the exact square root of 17424.



> **Note:** In a decimal number, the pairing of numbers starts from the decimal point. For the integral part it goes from right to left (←) and for the decimal part it goes from left to right, i.e., $\overleftarrow{31}\,\overleftarrow{61}.\overrightarrow{81}\,\overrightarrow{29}$. The procedure followed is the same as in integral numbers explained above.

(*v*) If a positive number is not a perfect square, then an approximate value of its square root may be obtained by the division method. $\sqrt{2}$ can be found as :



$\therefore\ \sqrt{2} = 1.414$ (approx.)

Also, if n is not a perfect square as 2, then $\sqrt{n}$ is not a rational number, e.g., $\sqrt{2}, \sqrt{3}, \sqrt{7}$ are not rational numbers.

4. The ***cube*** of a number is the *number raised to the power* 3, e.g., cube of $8 = 8^3 = 8 \times 8 \times 8 = 512$.

   A natural number n is a perfect cube if there exists a natural number m such that $n = m \times m \times m$, i.e., $m^3 = n$. 64 is a perfect cube as $64 = 4^3$.

5. (*i*) *The cube of an even number is even,* i.e., $6^3 = 216$.

   (*ii*) *The cube of an odd number is odd,* i.e., $5^3 = 125$.

6. The **cube root** of a number n is the number whose cube is n. It is denoted by $\sqrt[3]{n}$, e.g., $\sqrt[3]{8} = 2$.

7. The cube root of a number can be found by resolving the number into prime factors, making groups of 3 equal factors, picking out one of the equal factors from each group and multiplying the factors so picked.

   $1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

   $\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

8. The cube root of a negative perfect cube is negative, e.g., $\sqrt[3]{-125} = -5$

9. For any integer a and b, we have

   (*i*) $\sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$ (*ii*) $\sqrt[3]{\dfrac{a}{b}} = \dfrac{\sqrt[3]{a}}{\sqrt[3]{b}}$

QUESTION BANK

1. Find the value of $\sqrt{11.981 + 7\sqrt{1.2996}}$

   (a) 5.181 (b) 3.354
   (c) 4.467 (d) 4.924

2. What is the least number that must be added to 1901 so that the sum may be a perfect square is

   (a) 35 (b) 32
   (c) 30 (d) 29

3. The positive square root of 45.5625 is

   (a) 5.25 (b) 5.65
   (c) 6.35 (d) 6.75

4. The least perfect square number which is divisible by each of 21, 36 and 66 is

   (a) 213444 (b) 214344
   (c) 214434 (d) 231444

5. The square root of $0.09 + 2 \times 0.21 + 0.49$ is

(a) $\sqrt{0.09}+\sqrt{0.49}$ (b) $2\sqrt{0.21}$

(c) 1 (d) 0.58

6. The least integer that must be added to (9798×9792) to make it a perfect square is

(a) 9 (b) 8

(c) 7 (d) 6

7. Assume that $\sqrt{13}=3.605$ (approx.) and $\sqrt{130}$ = 11.40 (approx.).

Find the value of $\sqrt{1.3}+\sqrt{1300}+\sqrt{0.013}$

(a) 36.164 (b) 37.304

(c) 36.304 (d) 37.164

8. The digit in the units' place in the square root of 15876 is

(a) 8 (b) 6

(c) 4 (d) 2

9. The smallest number that must be added to 680621 to make the sum a perfect square is

(a) 4 (b) 5

(c) 6 (d) 8

10. $\sqrt{(0.798)^2+0.404\times 0.798+(0.202)^2}+1$ is equal to

(a) 0 (b) 2

(c) 1.596 (d) 0.404

11. What is the least number that should be subtracted from 0.000326 to have a perfect square is

(a) 0.000004 (b) 0.000002

(c) 0.04 (d) 0.02

12. Each member of a picnic party contributed twice as many rupees as the total number of members and the total collection was Rs 3042. The number of members present in the party was

(a) 2 (b) 32

(c) 40 (d) 39

13. What is the least number which must be subtracted from 10420 to make it a perfect square ?

(a) 219 (b) 200

(c) 189 (d) 16

14. $\sqrt{86.49}+\sqrt{5+k^2}=12.3$. So k is equal to

(a) $\sqrt{10}$ (b) $2\sqrt{5}$

(c) $3\sqrt{5}$ (d) 2

15. The number whose square is equal to the difference of the squares of 75.15 and 60.12 is

(a) 46.09 (b) 48.09

(c) 45.09 (d) 47.09

16. If $\sqrt{1369}+\sqrt{0.0615+x}=37.25$, then x is equal to

(a) 10^{-1} (b) 10^{-2}

(c) 10^{-3} (d) 10

17. If $\sqrt{(x-1)(y+2)}=7$, x and y being positive whole numbers, then the values of x and y are respectively

(a) 8, 5 (b) 15, 12

(c) 22, 19 (d) 6, 8

18. If $\sqrt{0.04\times 0.4\times a}=0.004\times 0.4\times\sqrt{b}$, then $\frac{a}{b}$ is

(a) 16×10^{-3} (b) 16×10^{-4}

(c) 16×10^{-5} (d) 16×10^{-2}

19. If $a = 0.1039$, then the value of $\sqrt{4a^2-4a+1}+3a$ is

(a) 0.1039 (b) 0.2078

(c) 1.1039 (d) 2.1039

20. If $3a = 4b = 6c$ and $a + b + c = 27\sqrt{29}$, then $\sqrt{a^2+b^2+c^2}$ is

(a) $3\sqrt{29}$ (b) 81

(c) 87 (d) 29

21. If $3\sqrt{5}+\sqrt{125}=17.88$, then what will be the value of $\sqrt{80}+6\sqrt{5}$?

(a) 13.41 (b) 20.46

(c) 21.66 (d) 22.35

22. The number of trees in each row of a garden is equal to the total number of rows in the garden. After 111 trees have been uprooted in a storm, their remain 10914 trees in the garden. The number of rows of trees in the garden is

(a) 100 (b) 105

(c) 115 (d) 125

23. If the product of four consecutive natural numbers increased by a natural number p is a perfect square, then the value of p is

(a) 8 (b) 4

(c) 2 (d) 1

24. $\sqrt[3]{\sqrt{0.000064}}$ is equal to

(a) 0.02 (b) 0.2

(c) 2 (d) 0.4

25. $\sqrt[2]{\sqrt[3]{x \times 0.000001}} = 0.2$. The value of x is

(a) 8 (b) 16

(c) 32 (d) 64

26. The digit in the units' place in the cube root of 21952 is

(a) 8 (b) 6

(c) 4 (d) 2

27. Cube root of a number when divided by 5 results in 25, what is the number ?

(a) 5 (b) 125^3

(c) 5^3 (d) 125

28. The smallest of $\sqrt{8}+\sqrt{5}$, $\sqrt{7}+\sqrt{6}$, $\sqrt{10}+\sqrt{3}$ and $\sqrt{11}+\sqrt{2}$ is

(a) $\sqrt{8}+\sqrt{5}$ (b) $\sqrt{7}+\sqrt{6}$

(c) $\sqrt{10}+\sqrt{3}$ (d) $\sqrt{11}+\sqrt{2}$

29. The smallest positive integer n for which $864 \times n$ is a perfect cube is

(a) 1 (b) 2

(c) 3 (d) 4

30. $\sqrt[3]{\sqrt[3]{a^3}}$ is equal to

(a) a (b) 1

(c) $a^{1/3}$ (d) a^3

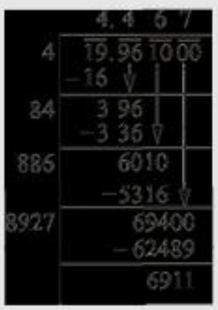
Answers

1. (c)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (b)
11. (b)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (c)
21. (d)	**22.** (b)	**23.** (d)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (b)	**30.** (c)

Hints and Solutions

1. (c) $\sqrt{11.981+7\sqrt{1.2996}} = \sqrt{11.981+7\times 1.14}$

$= \sqrt{11.981+7.98} = \sqrt{19.961} = 4.468$



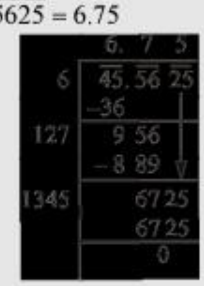
2. (a) Number to be added

$= (44)^2 - 1901$

$= 1936 - 1901$

$= 35$

3. (d) $\therefore \sqrt{45.5625} = 6.75$



4. (b) LCM of 21, 36 and 66 = 2772

3	21, 36, 66
2	7, 12, 22
	7, 6, 11

$\therefore$ Least perfect square number

$= (2^2 \times 3^2 \times 7 \times 11) \times 7 \times 11 = 213444$

5. (c) $\sqrt{0.09 + 2\times 0.21 + 0.49}$

$= \sqrt{(0.3)^2 + 2\times 0.3\times 0.7 + (0.7)^2}$

$= \sqrt{(0.3+0.7)^2} = \sqrt{1} = 1$.

6. (a) $9798 \times 9792 = (9792 + 6) \times 9792$

$= (9792)^2 + 6 \times 9792$

$= (9792)^2 + 2 \times 3 \times 9792$

$\therefore$ Perfect square number

$= (9792)^2 + 2 \times 3 \times 9792 + (3)^2$

$\therefore$ Least integer to be added to make 9798×9792 a perfect square $= 3^2 = 9$.

7. (b) $\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013}$

$= \sqrt{\frac{130}{100}} + \sqrt{13\times 100} + \sqrt{\frac{130}{10000}}$

$= \frac{\sqrt{130}}{\sqrt{100}} + \sqrt{13}\times\sqrt{100} + \frac{\sqrt{130}}{\sqrt{10000}}$

$= \frac{11.40}{10} + 3.605 \times 10 + \frac{11.40}{100}$

$= 1.14 + 36.05 + 0.114 = 37.304$

8. (b)

$\therefore \quad \sqrt{15876} = 126$

$\Rightarrow$ Digit in units' place in $\sqrt{15876} = 6$.

9. (a)

$\therefore$ Least number to be added to 680621 to make the sum a perfect square
$= (825)^2 - 680621$
$= 680625 - 680621 = 4.$

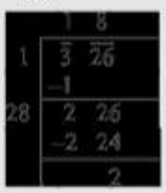
10. (b) Given exp.

$= \sqrt{(0.798)^2 + 2 \times 0.202 \times 0.798 + (0.202)^2} + 1$

$= \sqrt{(0.798 + 0.202)^2} + 1$

$= \sqrt{1^2} + 1 = 1 + 1 = 2.$

11. (b) $0.000326 = \frac{326}{10^6}$



$\therefore$ Least number to be subtracted $= \frac{2}{10^6} = 0.000002$

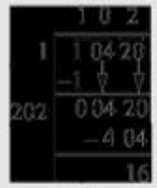
12. (d) Let the total number of members be x. Then,
Each member's contribution = Rs $2x$
Given, $x \times 2x = 3042$

$\Rightarrow 2x^2 = 3042$

$\Rightarrow x^2 = 1521$

$\Rightarrow x = 39.$

13. (d)



$\therefore$ Least number to be subtracted from 10420 to make it a perfect square = 16.

14. (d) $\sqrt{86.49} + \sqrt{5 + k^2} = 12.3$

$\Rightarrow \sqrt{5 + k^2} = 12.3 - \sqrt{86.49} = 12.3 - 9.3 = 3$

$\Rightarrow 5 + k^2 = 9$ (on squaring both the sides)

$\Rightarrow k^2 = 9 - 5 = 4 \Rightarrow k = 2$

15. (c) Let the required number be x. Then.
$x^2 = (75.15)^2 - (60.12)^2$
$= (75.15 + 60.12)\,(75.15 - 60.12)$
$= 135.27 \times 15.03 = 2033.1081$

$\therefore \quad x = \sqrt{2033.1081}$
$= 45.09.$

16. (c) $\sqrt{1369} + \sqrt{0.0615 + x} = 37.25$

$\Rightarrow \sqrt{0.0615 + x} = 37.25 - \sqrt{1369}$
$= 37.25 - 37 = 0.25$

$\Rightarrow 0.0615 + x = (0.25)^2 = 0.0625$

$\Rightarrow x = 0.0625 - 0.0615 = 0.001 = \frac{1}{10^3} = 10^{-3}.$

17. (a) $\sqrt{(x-1)(y+2)} = 7 \Rightarrow (x-1)\,(y+2) = 7^2$

$\Rightarrow (x - 1) = 7$ and $(y + 2) = 7$

$\Rightarrow x = 8$ and $y = 5$.

18. (c) Given exp. $\Rightarrow \sqrt{0.016 \times a} = 0.0016 \times \sqrt{b}$

$\Rightarrow \sqrt{0.016} \times \sqrt{a} = 0.0016 \times \sqrt{b} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{0.0016}{\sqrt{0.016}}$

$\Rightarrow \frac{a}{b} = \frac{0.0016 \times 0.0016}{0.016} = 0.00016$

$= \frac{16}{100000} = 16 \times 10^{-5}.$

19. (c) $\sqrt{4a^2 - 4a + 1} + 3a$

$= \sqrt{(1)^2 - 2\times 2a\times 1 + (2a)^2} + 3a$

$= \sqrt{(1-2a)^2} + 3a = (1-2a) + 3a = 1 + a$

$= 1 + 0.1039 = 1.1039.$

20. (c) $3a = 4b = 6c \Rightarrow 4b = 6c \Rightarrow b = \frac{3}{2}c$

and $3a = 4b \Rightarrow a = \frac{4}{3}b = \frac{4}{3}\times\frac{3}{2}c = 2c$

$\therefore\ a + b + c = 27\sqrt{29}$

$\Rightarrow 2c + \frac{3}{2}c + c = 27\sqrt{29}$

$\Rightarrow \frac{9}{2}c = 27\sqrt{29} \Rightarrow c = 6\sqrt{29}$

Now, $\sqrt{a^2 + b^2 + c^2}$

$= \sqrt{(a+b+c)^2 - 2(ab + bc + ca)}$

$= \sqrt{(27\sqrt{29})^2 - 2\left(2c\times\frac{3}{2}c + \frac{3}{2}c\times c + c\times 2c\right)}$

$= \sqrt{729\times 29 - 2\left(3c^2 + \frac{3}{2}c^2 + 2c^2\right)}$

$= \sqrt{729\times 29 - 2\times\frac{13c^2}{2}}$

$= \sqrt{729\times 29 - 13\times(6\sqrt{29})^2}$

$= \sqrt{29(729 - 468)} = \sqrt{29\times 261} = \sqrt{29\times 29\times 9}$

$= 29\times 3 = 87.$

21. (d) $3\sqrt{5} + \sqrt{125} = 17.88$

$\Rightarrow 3\sqrt{5} + \sqrt{5^2\times 5} = 17.88$

$\Rightarrow 3\sqrt{5} + 5\sqrt{5} = 17.88$

$\Rightarrow 8\sqrt{5} = 17.88 \Rightarrow \sqrt{5} = \frac{17.88}{8} = 2.235$

Now, $\sqrt{80} + 6\sqrt{5} = \sqrt{16\times 5} + 6\sqrt{5}$

$= 4\sqrt{5} + 6\sqrt{5} = 10\sqrt{5}$

$= 10\times 2.235 = 22.35.$

22. (b) Let the number of rows = number of trees = x

$\therefore$ Total number of trees in the garden

$= x\times x = x^2 = 10914 + 111 = 11025$

$\therefore$ No. of rows of trees = $\sqrt{11025} = 105$

23. (d) Let the four consecutive natural numbers be $x,\ x+1,\ x+2$ and $x+3$. Then,

A perfect square $= x(x+1)(x+2)(x+3) + p$

$= x(x+3)(x+1)(x+2) + p$

$= (x^2 + 3x)\times(x^2 + 3x + 2) + p$

$= (x^2 + 3x)^2 + 2(x^2 + 3x + 2) + p$

$= (x^2 + 3x)^2 + 2(x^2 + 3x) + p$

The expression on the right hand side will be a perfect square if and only $p = 1$.

Perfect square number

$= [(x^2 + 3x)^2 + 2(x^2 + 3x) + 1]$

$= (x^2 + 3x + 1)^2$

24. (b) $\sqrt[3]{\sqrt{0.000064}} = \sqrt[3]{\sqrt{\frac{64}{1000000}}}$

$= \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0.2$

25. (*d*) $\sqrt[2]{\sqrt[3]{x\times 0.000001}} = 0.2$

$\Rightarrow \sqrt[3]{x\times 0.000001} = (0.2)^2$

$= 0.04$ (Squaring both the sides)

$\Rightarrow x\times 0.000001 = (0.04)^3 = 0.000064$

(on taking the cube of both the sides)

$\Rightarrow x = \frac{0.000064}{0.000001} = 64.$

26. (a)

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
	7

$\therefore\ 21952 = 2\times 2\times 2\times 2\times 2\times 2\times 7\times 7\times 7$

$= 2^3\times 2^3\times 7^3$

$\therefore\ \sqrt[3]{21952} = \sqrt[3]{2^3\times 2^3\times 7^3} = 2\times 2\times 7 = 28$

$\therefore$ Digit in the units' place of $\sqrt[3]{21952} = 8$.

27. (b) Let the number be x. Then,

$\frac{\sqrt[3]{x}}{5} = 25 \Rightarrow \sqrt[3]{x} = 125 \Rightarrow x = (125)^3$

28. (d) $\sqrt{8} + \sqrt{5} = 2.83 + 2.24 = 5.07$

$\sqrt{7} + \sqrt{6} = 2.65 + 2.45 = 5.09$

$\sqrt{10} + \sqrt{13} = 3.16 + 3.61 = 6.77$

$\sqrt{11} + \sqrt{2} = 3.32 + 1.41 = 4.73$

$\therefore$ Smallest is $\sqrt{11} + \sqrt{2}$

29. (b)

2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$864 = \underline{2 \times 2 \times 2} \times 2 \times 2 \times \underline{3 \times 3 \times 3}$

Since 2×2 is the only in complete triplet, so 864 has to be multiplied by 2 to make it a perfect cube.

$\Rightarrow n = 2$.

30. (c) $\sqrt[3]{\sqrt[3]{a^3}} = \left((a^3)^{\frac{1}{3}}\right)^{\frac{1}{3}} = a^{3\times\frac{1}{9}} = a^{\frac{1}{3}}$.

SELF ASSESSMENT EXERCISE

1. If $\sqrt{(x-1)(y+2)} = 7$ and x and y are positive whole numbers, their values respectively are

(a) 8, 5 (b) 15, 12

(c) 22, 19 (d) None of these

2. The square root of the expression

$$\frac{(12.1)^2 - (8.1)^2}{(0.25)^2 + (0.25)(19.95)} \text{ is}$$

(a) 1 (b) 2

(c) 3 (d) 4

3. Consider the following values of the three given numbers: $\sqrt{103}$, $\sqrt{99.35}$, $\sqrt{102.20}$

1. 10.1489 (approx.)

2. 10.109 (approx.)

3. 9.967 (approx.)

The correct sequence of these values matching with the above numbers is:

(a) 1, 2, 3 (b) 1, 3, 2

(c) 2, 3, 1 (d) 3, 1, 2

4. What value should come in place of the question mark (?) in the following equation?

$$48\sqrt{?} + 32\sqrt{?} = 320$$

(a) 16 (b) 2

(c) 4 (d) 32

5. Which is greater $(\sqrt{7} + \sqrt{10})$ or $(\sqrt{3} + \sqrt{19})$?

(a) $\sqrt{7} + \sqrt{10}$ (b) $\sqrt{3} + \sqrt{19}$

(c) both are equal (d) none of these

6. Find the least number which if added to 17420 will make it a perfect square?

(a) 3 (b) 5

(c) 9 (d) 4

7. Calculate the value of N in the given series and then find the value of x using the given equation.

99 163 N 248 273 289

If $\sqrt{2N + 17} = x$, then x equals

(a) 20.5 (b) 20.0

(c) 21.5 (d) 21.0

8. The largest number of 5-digits that is a perfect square is

(a) 99900 (b) 99856

(c) 99981 (d) 99801

9. If $99 \times 21 - \sqrt[3]{x} = 1968$, then x equals

(a) 1367631 (b) 1366731

(c) 1367 (d) 111

10. If $P = 999$, then $\sqrt[3]{P(P^2 + 3P + 3) + 1} =$

(a) 1000 (b) 999

(c) 1002 (d) 998

Answers

1. (a)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (a)