



# ST. LAWRENCE HIGH SCHOOL



A JESUIT CHRISTIAN MINORITY INSTITUTION

CLASS 8

SUBJECT :Algebra & Geometry

STUDY MATERIAL 6

Properties of Triangles

Date:9.5.2020

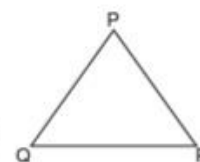
## PROPERTIES OF TRIANGLES

### KEY FACTS

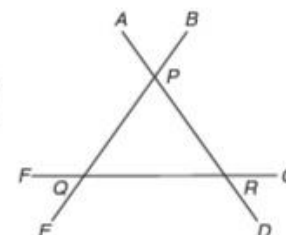
#### Definitions

A triangle is a three sided closed figure formed by three non-collinear points.

The three points  $P$ ,  $Q$  and  $R$  in the given figure are called the **vertices**, line segments joining the three vertices, i.e.,  $PQ$ ,  $QR$  and  $PR$  are called the **sides** and  $\angle P$ ,  $\angle Q$  and  $\angle R$  are the **interior angles** of the triangle.



If the sides of a triangle are produced as shown in the given diagram, then the angles  $\angle PRC$ ,  $\angle QRD$ ,  $\angle PFQ$ ,  $\angle RQE$ ,  $\angle QPA$  and  $\angle RPB$  are the exterior angles of  $\triangle ABC$ .



#### Types of Triangles:

##### a. By sides:

<p><b>Scalene Triangle</b></p> <p><math>a \neq b \neq c</math> (All the sides are unequal)</p>	<p><b>Isosceles Triangle</b></p> <p>(At least two sides are equal. Here, <math>AB = AC</math>) <b>Angles opposite equal sides are also equal, i.e.,</b> <math>\angle C = \angle B</math>.</p>	<p><b>Equilateral Triangle</b></p> <p><math>a = b = c</math> (All sides are equal) <b>All angles are equal to <math>60^\circ</math></b></p>
--	---	---

##### b. By angles:

<p><b>Acute Angled Triangle</b></p> <p>All angles are acute, i.e., <math>\angle A &lt; 90^\circ</math>, <math>\angle B &lt; 90^\circ</math>, <math>\angle C &lt; 90^\circ</math></p>	<p><b>Right Angled Triangle</b></p> <p>One of the angles is a right angle. The other two are <b>complementary</b> to each other</p>	<p><b>Obtuse Angled Triangle</b></p> <p>One of the angles is an obtuse angle.</p>
--	---	---

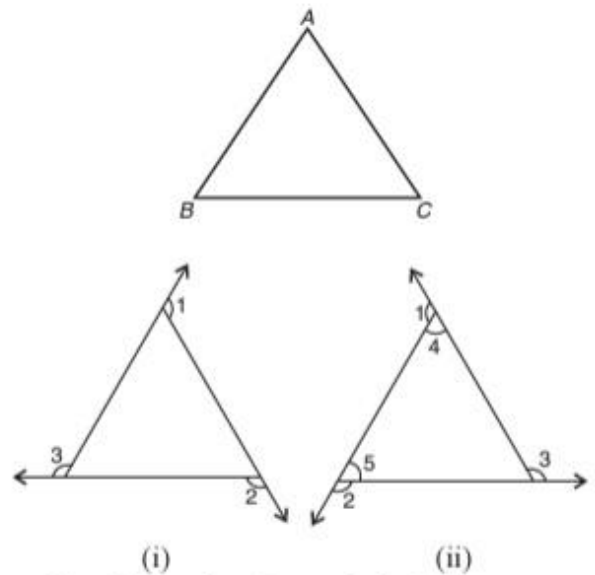
## Some Important Properties of Triangles:

a. *The sum of the three interior angles of a triangle is always  $180^\circ$ , i.e.,  $\angle BAC + \angle ABC + \angle BCA = 180^\circ$ .*

b. (i) *If the sides of a triangle are produced in order then, the sum of the three (ordered) exterior angles of a triangle is  $360^\circ$ , i.e., in both the figures,  $\angle 1 + \angle 2 + \angle 3 = 360^\circ$*

(ii) *The measure of an exterior angle is equal to the sum of the measures of the interior opposite angles, i.e., in figure (ii)  $\angle 3 = \angle 4 + \angle 5$ .*

(iii) *The measure of an exterior angle is greater than the measure of each of the interior opposite angles, i.e., in figure (ii)  $\angle 3 > \angle 4$  and  $\angle 3 > \angle 5$ .*



(iv) *The sum of the measure of exterior angle at a vertex and its adjacent interior angle is  $180^\circ$ .*

## Triangle Inequalities:

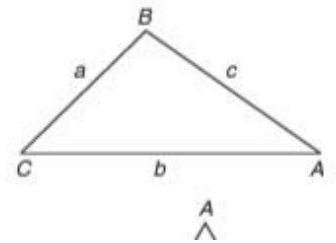
(i) *Sum of any two sides of a triangle is always greater than the third side.*

(ii) *The difference of any two sides is always less than the third side.*

(iii) *If two sides of a triangle are not equal, then the angle opposite to the greater side is greater and vice versa.*

(iv) Let  $a, b, c$  be the three sides of a triangle  $\triangle ABC$  where  $AB = c$  is the longest side (say). Then,

- if  $c^2 < a^2 + b^2$ , then the triangle is acute angled.
- if  $c^2 = a^2 + b^2$ , then the triangle is right angled.
- if  $c^2 > a^2 + b^2$ , then the triangle is obtuse angled.



1. A **triangle** is a plane closed figure bounded by three line segments.

(i) Sum of the angles of a triangle is equal to  $180^\circ$ , i.e.,

$$\angle A + \angle B + \angle C = 180^\circ$$

(ii) Exterior angle of a triangle is equal to the sum of its interior opposite angles, i.e.,

$$\angle x = \angle 2 + \angle 3; \angle y = \angle 1 + \angle 2; \angle z = \angle 1 + \angle 3$$

(iii) Sum of any two sides of a triangle is always, greater than the third side, i.e.,

$$PQ + QR > PR$$

$$PQ + PR > QR$$

$$PR + QR > PQ$$

(iv) Side opposite to the greatest angle will be greatest in length and vice versa.

2. **Important Terms of a Triangle**

(i) **Median and centroid:** A line joining the mid-point of a side of a triangle to its opposite vertex is called the median.  $D, E, F$  are the mid-points of the sides  $QR, PR$  and  $PQ$  respectively of a  $\triangle PQR$ . Then,  $PD, QE$  and  $RF$  are the medians of  $\triangle PQR$ .

- The point of concurrency of the three medians of a triangle is called **centroid**.

- The centroid of a triangle divides each median in the ratio 2:1, i.e.,

$$PG : GD = QG : GE = RG : GF = 2:1$$

- A median divides a triangle into two parts of equal area.

(ii) **Perpendicular bisector and circumcentre:** Perpendicular bisector to any side is the line that is perpendicular to that side and passes through its mid-point. Perpendicular bisectors need not pass through the opposite vertex.

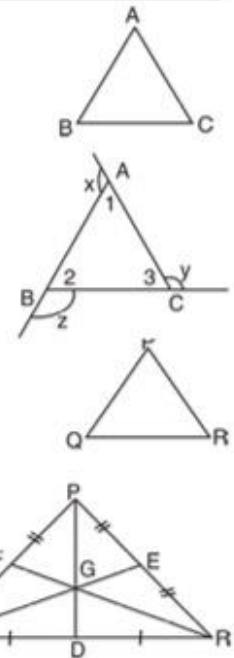
- The point of intersection of the three perpendicular bisectors of a triangle is called its **circumcentre**.

- The **circumcentre** of a triangle is **equidistant from its three vertices**.

If we draw a circle with circumcentre as the centre and the distance of any vertex from the circumcentre as radius, the circle passes through all the three vertices and the circle is called **circumcircle**.

**Note.** The circumcentre can be inside or outside the circle.

- Circumcentre of a right angled triangle is the mid-point of the hypotenuse.



(iii) **Angle bisector and in-centre:**

- The point of intersection of the three angle bisectors of a triangle is called its **in-centre**.
- The **in-centre** always lies inside the triangle.
- It is always **equidistant from the sides of a triangle**.
- The circle drawn with incentre as centre and touching all the three sides of a triangle is called **in-circle**.



(iv) **Altitude and ortho-centre:**

The perpendicular drawn from the vertex of a triangle to the opposite side is called an **altitude**.

- The point of intersection of the three altitudes of a triangle is called **ortho-centre**, which can lie inside or outside the triangle.

**Note.**

- For an isosceles triangle, the median drawn from a vertex to the opposite side is also the perpendicular bisector of that side.
- In an equilateral triangle, the median, angle bisector, altitude and perpendicular bisector of sides are all represented by the same straight line.
- The circumcentre, centroid, orthocentre and incentre all coincide in an equilateral triangle.

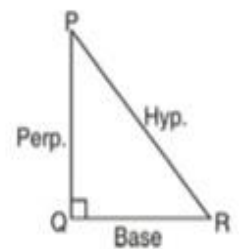
**3. Pythagoras' theorem:**

- (i) In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$\therefore (\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$$

$$\Rightarrow PQ^2 + QR^2 = PR^2$$

A



## ENRICHMENT

### Pythagorean Triplets

The Pythagorean Property relates the lengths,  $a$  and  $b$ , of the two legs of a right triangle with the length  $c$  of the hypotenuse by the equation :  $a^2 + b^2 = c^2$ .

**If three natural numbers  $a$ ,  $b$  and  $c$  are related so that  $a^2 + b^2 = c^2$  then  $a$ ,  $b$  and  $c$  are called a Pythagorean triplet.**

Thus, 5, 12 and 13 are a Pythagorean triplets because  $5^2 + 12^2 = 13^2$ . 8, 9 and 12 are not a Pythagorean triplets because  $8^2 + 9^2 \neq 12^2$ .

**Note.** You can show that if  $n$  is any positive real number, then,  $3n$ ,  $4n$  and  $5n$  represent sides of a right triangle

$$(5n)^2 = 25n^2$$

$$(3n)^2 + (4n)^2 = 9n^2 + 16n^2 = 25n^2$$

Therefore,

$$(5n)^2 = (3n)^2 + (4n)^2$$

In general, **if  $(a, b, c)$  is a Pythagorean triplet and  $k$  is any positive number, then  $ak$ ,  $bk$  and  $ck$  represent the three sides of a right triangle.**

**Note.** One method of obtaining some Pythagorean triplets is to choose two natural numbers  $m$  and  $n$  so that  $m > n$ , and taking

$$a = m^2 - n^2, b = 2mn, c = m^2 + n^2$$

For example,

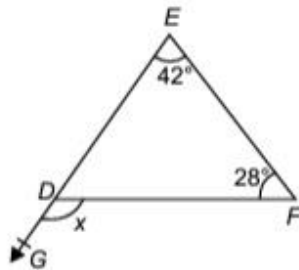
$m$	$n$	$2mn$	$m^2 - n^2$	$m^2 + n^2$
3	2	12	5	13
4	3	24	7	25

12, 5, 13 and 24, 7, 25 are Pythagorean triplets.

# QUESTION BANK

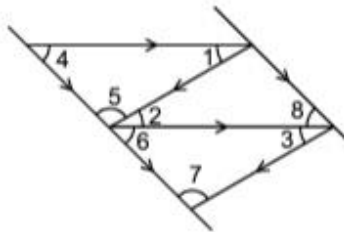
## MATHEMATICAL REASONING

1. Find the measure of the angle  $x$  in the given figure.



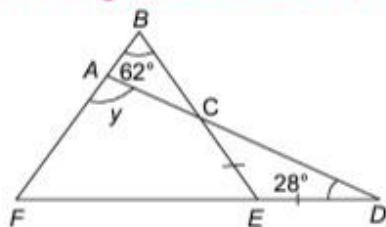
- (A)  $50^\circ$  (B)  $70^\circ$  (C)  $60^\circ$  (D)  $30^\circ$

2. Which of the following options is INCORRECT?



- (A)  $\angle 1 = \angle 3$   
 (B)  $\angle 1 + \angle 4 + \angle 5 = 180^\circ$   
 (C)  $\angle 8 = \angle 6$   
 (D)  $\angle 1 + \angle 3 = 180^\circ$

3. In the figure (not drawn to scale),  $ADF$  and  $BEF$  are triangles and  $EC = ED$ , find  $y$ .



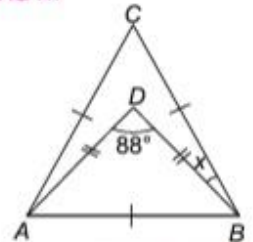
- (A)  $90^\circ$  (B)  $91^\circ$  (C)  $92^\circ$  (D)  $93^\circ$

4. In a  $\triangle ABC$ , which of the given conditions holds ?

- (A)  $AB - BC > CA$  (B)  $AB + BC < CA$   
 (C)  $AB - BC < CA$  (D)  $AB + CA < BC$

5. In the figure (not drawn to scale),  $ABC$  is an equilateral triangle and  $ABD$  is an isosceles triangle with  $DA = DB$ , find  $x$ .

- (A)  $14^\circ$   
 (B)  $16^\circ$   
 (C)  $12^\circ$   
 (D)  $32^\circ$

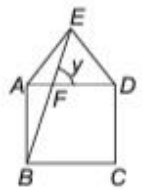


6. If  $ABC$  is an isosceles triangle with  $AB = AC$  and  $AD$  is an altitude, then \_\_\_\_\_.

- (A)  $\angle B > \angle C$  (B)  $\angle B < \angle C$   
 (C)  $\angle B = \angle C$  (D) None of these

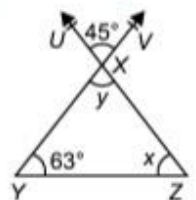
7. In the figure (not drawn to scale),  $ABCD$  is a square,  $ADE$  is an equilateral triangle and  $BFE$  is a straight line, find  $y$ .

- (A)  $90^\circ$   
 (B)  $45^\circ$   
 (C)  $75^\circ$   
 (D)  $15^\circ$



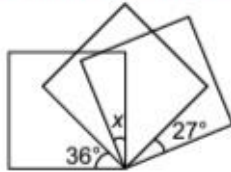
8. Find the measure of the angle  $x$  in the given figure.

- (A)  $72^\circ$   
 (B)  $82^\circ$   
 (C)  $90^\circ$   
 (D)  $40^\circ$

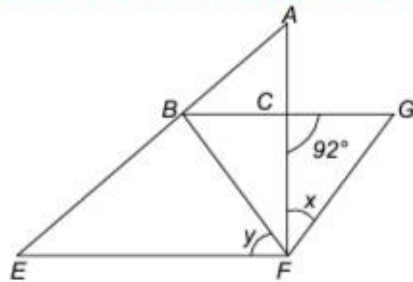


9. The given figure shows three identical squares. Find  $x$ .

- (A)  $30^\circ$   
 (B)  $27^\circ$   
 (C)  $36^\circ$   
 (D)  $16^\circ$

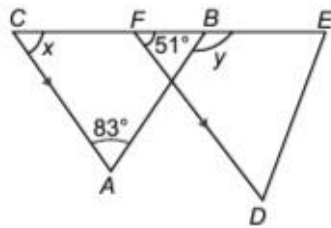


10. In the figure (not drawn to scale),  $EFA$  is a right-angled triangle with  $\angle EFA = 90^\circ$  and  $FGB$  is an equilateral triangle, find  $y - 2x$ .



- (A)  $2^\circ$  (B)  $8^\circ$  (C)  $17^\circ$  (D)  $20^\circ$

11. In the figure (not drawn to scale),  $ABC$  and  $DEF$  are two triangles,  $CA$  is parallel to  $FD$  and  $CFBE$  is a straight line. Find the value of  $x + y$ .



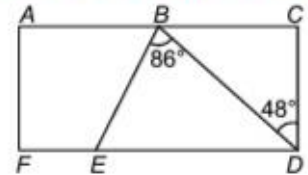
- (A)  $185^\circ$  (B)  $134^\circ$  (C)  $148^\circ$  (D)  $176^\circ$

12. In a  $\triangle ABC$ , if  $AB + BC = 10$  cm,  $BC + CA = 12$  cm,  $CA + AB = 16$  cm, then the perimeter of the triangle is \_\_\_\_\_.

- (A) 19 cm (B) 17 cm  
 (C) 28 cm (D) 22 cm

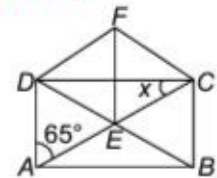
13. In the figure, not drawn to scale,  $ACDF$  is a rectangle and  $BDE$  is a triangle. Find  $\angle BED$ .

- (A)  $42^\circ$   
 (B)  $52^\circ$   
 (C)  $128^\circ$   
 (D)  $134^\circ$



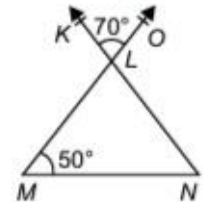
14. In the figure,  $ABCD$  is a rectangle,  $\triangle CEF$  is an equilateral triangle. Find  $x$ .

- (A)  $25^\circ$   
 (B)  $30^\circ$   
 (C)  $20^\circ$   
 (D)  $50^\circ$



15. Find the measure of  $\angle LNM$  in the given figure.

- (A)  $30^\circ$   
 (B)  $80^\circ$   
 (C)  $70^\circ$   
 (D)  $60^\circ$



## EVERYDAY MATHEMATICS

16. A 26 m long ladder reached a window 24 m from the ground on placing it against a wall. Find the distance of the foot of the ladder from the wall.

- (A) 10 m (B) 20 m  
 (C) 5 m (D) 25 m

17. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

- (A) 20 m (B) 36 m  
 (C) 18 m (D) 25 m

18. Aryan wants to plant a flower on the ground in the form of a rhombus. The diagonals of the rhombus measures 42 cm and 56 cm. Find the perimeter of the field.

- (A) 150 cm (B) 140 cm  
 (C) 130 cm (D) 120 cm

19. A 34 m long ladder reached a window 16 m from the ground on placing it against a wall. Find the distance of the foot of the ladder from the wall.

- (A) 40 m (B) 30 m  
 (C) 50 m (D) 10 m

20. Mrs Kaushik gives a problem to her students. Find the perimeter of a rectangle whose length is 28 cm and diagonal is 35 cm.

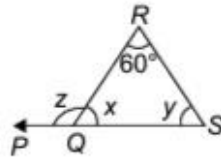
What will be the correct answer?

- (A) 90 cm (B) 45 cm  
(C) 89 cm (D) 98 cm

### ACHIEVERS SECTION (HOTS)

21. If  $y$  is five times of  $x$ , find the values of  $x$ ,  $y$  and  $z$ .

- |     |            |             |             |
|-----|------------|-------------|-------------|
|     | <b>x</b>   | <b>y</b>    | <b>z</b>    |
| (A) | $20^\circ$ | $80^\circ$  | $140^\circ$ |
| (B) | $30^\circ$ | $80^\circ$  | $140^\circ$ |
| (C) | $20^\circ$ | $100^\circ$ | $160^\circ$ |
| (D) | $30^\circ$ | $100^\circ$ | $160^\circ$ |



22. State 'T' for true and 'F' for false.

- (i) In the given right-angled triangle  $ABC$ ,  $\angle B = 65^\circ$ ,  $\angle C = 25^\circ$ , then  $AB^2 = BC^2 + CA^2$ .
- (ii) The length of the third side of a triangle cannot be smaller than the difference of the lengths of the other two sides.
- (iii) A triangle can have only one median.

- |     |            |             |              |
|-----|------------|-------------|--------------|
|     | <b>(i)</b> | <b>(ii)</b> | <b>(iii)</b> |
| (A) | F          | F           | T            |
| (B) | F          | T           | F            |
| (C) | F          | T           | T            |
| (D) | F          | F           | F            |

23. Fill in the blanks.

- (i) The line segment joining a vertex of a triangle to the midpoint of its opposite side is called a/an P of the triangle.
- (ii) The perpendicular line segment

from a vertex of a triangle to its opposite side is called a/an Q of the triangle.

- (iii) A triangle has R altitudes and S medians.

- |     |          |          |          |          |
|-----|----------|----------|----------|----------|
|     | <b>P</b> | <b>Q</b> | <b>R</b> | <b>S</b> |
| (A) | Altitude | Median   | 1        | 1        |
| (B) | Altitude | Median   | 3        | 3        |
| (C) | Median   | Altitude | 3        | 3        |
| (D) | Median   | Altitude | 2        | 3        |

24. Which of the following statements is TRUE?

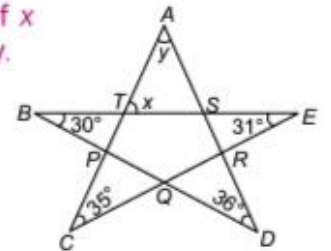
**Statement-1** : The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

**Statement-2** : If  $P$  is a point on the side  $BC$  of  $\triangle ABC$ . Then  $(AB + BC + AC) > 2AP$

- (A) Only Statement-1  
(B) Only Statement-2  
(C) Both Statement-1 and Statement-2  
(D) Neither Statement-1 nor Statement-2

25. Find the values of  $x$  and  $y$  respectively.

- (A)  $47^\circ, 66^\circ$   
(B)  $66^\circ, 48^\circ$   
(C)  $68^\circ, 47^\circ$   
(D)  $47^\circ, 68^\circ$



Darken your choice with HB Pencil

1. (A) (B) (C) (D)	8. (A) (B) (C) (D)	15. (A) (B) (C) (D)	22. (A) (B) (C) (D)
2. (A) (B) (C) (D)	9. (A) (B) (C) (D)	16. (A) (B) (C) (D)	23. (A) (B) (C) (D)
3. (A) (B) (C) (D)	10. (A) (B) (C) (D)	17. (A) (B) (C) (D)	24. (A) (B) (C) (D)
4. (A) (B) (C) (D)	11. (A) (B) (C) (D)	18. (A) (B) (C) (D)	25. (A) (B) (C) (D)
5. (A) (B) (C) (D)	12. (A) (B) (C) (D)	19. (A) (B) (C) (D)	
6. (A) (B) (C) (D)	13. (A) (B) (C) (D)	20. (A) (B) (C) (D)	
7. (A) (B) (C) (D)	14. (A) (B) (C) (D)	21. (A) (B) (C) (D)	



# SOLUTIONS

1. (B) :  $\angle EFD + \angle FED = x$   
 (Exterior angle property of a triangle)  
 $\Rightarrow 28^\circ + 42^\circ = \angle x$   
 or  $\angle x = 70^\circ$

2. (D) :  $\angle 1 = \angle 2$  and  $\angle 2 = \angle 3$  [Alternate angles]  
 So,  $\angle 1 = \angle 3$   
 and  $\angle 1 + \angle 4 + \angle 5 = 180^\circ$  [Angle sum property]  
 Also,  $\angle 8 = \angle 6$  [Alternate angles]

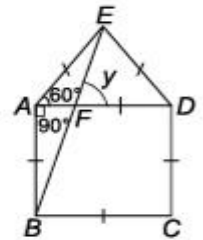
3. (A) : In  $\triangle CED$ ,  
 $CE = ED$   
 $\therefore \angle EDC = \angle ECD$   
 [Angles opposite to equal sides are equal]  
 $\Rightarrow \angle ECD = 28^\circ$   
 Also,  $\angle ECD = \angle BCA$  (Vertically opposite angles)  
 $\Rightarrow \angle BCA = 28^\circ$   
 In  $\triangle BCA$ ,  
 $y = 62^\circ + 28^\circ$  [Exterior angle property]  
 $\Rightarrow y = 90^\circ$

4. (C)

5. (A) : Since  $ABC$  is an equilateral triangle.  
 $\therefore \angle CAB = \angle ABC = \angle BCA = 60^\circ$   
 And  $\angle DBA = \angle DAB = (60^\circ - x)$  [ $\because DA = DB$ ]  
 In  $\triangle DAB$ ,  
 $\angle DBA + \angle DAB + \angle ADB = 180^\circ$   
 $\Rightarrow 2(60^\circ - x) + 88^\circ = 180^\circ$   
 $\Rightarrow 2(60^\circ - x) = 92^\circ \Rightarrow 60^\circ - x = 46^\circ \Rightarrow x = 14^\circ$

6. (C) :

7. (C) : In  $\triangle AEB$ ,  
 $\angle A = \angle DAE + \angle BAD$   
 $\Rightarrow \angle A = 60^\circ + 90^\circ = 150^\circ$   
 And,  $AE = AB$   
 $\Rightarrow \angle ABE = \angle AEB$

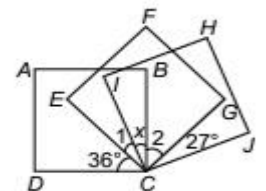


[Angles opposite to equal sides are equal]

- Now,  $\angle A + \angle ABE + \angle AEB = 180^\circ$   
 (Angle sum property)  
 $\Rightarrow 2\angle AEB = 180^\circ - 150^\circ = 30^\circ \Rightarrow \angle AEB = 15^\circ$   
 Now,  $\angle E = 60^\circ$   
 $\Rightarrow \angle DEF = 60^\circ - 15^\circ = 45^\circ$   
 $\therefore$  In  $\triangle EFD$ ,  
 $\angle DEF + \angle EDF + \angle EFD = 180^\circ$   
 $\Rightarrow 45^\circ + 60^\circ + y = 180^\circ$   
 $\Rightarrow y = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$

8. (A) :  $\angle UXV = y$  (Vertically opposite angles)  
 $\therefore y = 45^\circ$   
 In  $\triangle XYZ$   
 $y + x + 63^\circ = 180^\circ$  (Angle sum property)  
 $\Rightarrow 45^\circ + x + 63^\circ = 180^\circ \Rightarrow x = 180^\circ - (45^\circ + 63^\circ)$   
 $\Rightarrow x = 180^\circ - 108^\circ = 72^\circ$

9. (B) : We have,  $ABCD$ ,  
 $CEFG$  and  $CIHJ$  are all squares.  
 So,  $\angle 1 + \angle 2 + x = 90^\circ \dots$  (i)  
 $36^\circ + \angle 1 + x = 90^\circ \dots$  (ii)  
 $x + \angle 2 + 27^\circ = 90^\circ \dots$  (iii)  
 Adding (ii) and (iii), we get



$$36^\circ + x + 27^\circ + (\angle 1 + \angle 2 + x) = 180^\circ$$

$$\Rightarrow 63^\circ + x + 90^\circ = 180^\circ \text{ (From (i))}$$

$$\Rightarrow x = 180^\circ - 153^\circ = 27^\circ$$

10. (A) : In  $\triangle FGC$ ,  $\angle GCF = 92^\circ$  (given)  
As we know,  $\angle CGF = 60^\circ$   
(Angle of equilateral triangle)  
 $\therefore x + 60^\circ + 92^\circ = 180^\circ$   
 $\Rightarrow x = 180^\circ - 152^\circ = 28^\circ$   
Now, in  $\triangle BCF$ ,  $\angle CBF = 60^\circ$   
 $\angle FCB = 180^\circ - 92^\circ$  (Linear pair)  
 $\Rightarrow \angle FCB = 88^\circ$   
 $\therefore \angle BFC + 88^\circ + 60^\circ = 180^\circ$   
(Angle sum property)  
 $\Rightarrow \angle BFC = 180^\circ - 148^\circ = 32^\circ$   
And,  $\angle AFE = 90^\circ$   
 $\Rightarrow y + 32^\circ = 90^\circ \Rightarrow y = 90^\circ - 32^\circ = 58^\circ$   
 $\therefore y - 2x = 58^\circ - 2 \times 28^\circ = 58^\circ - 56^\circ = 2^\circ$

11. (A) :  $\angle FCA = \angle BFD$  (Corresponding angles)  
 $\Rightarrow x = 51^\circ$   
Now, in  $\triangle ABC$   
 $y = 51^\circ + 83^\circ$  (Exterior angle property)  
 $\Rightarrow y = 134^\circ$   
So,  $x + y = 51^\circ + 134^\circ = 185^\circ$

12. (A) : It is given that,  
 $AB + BC = 10$  cm ... (i)  
 $BC + CA = 12$  cm ... (ii)  
 $CA + AB = 16$  cm ... (iii)  
Adding (i), (ii) and (iii); we get,  
 $2(AB + BC + CA) = 10 + 12 + 16$   
 $\Rightarrow AB + BC + CA = 19$  cm.

13. (B) :  $\angle CDB + \angle BDE = 90^\circ$  (Angle of a rectangle)  
 $\Rightarrow 48^\circ + \angle BDE = 90^\circ$   
 $\Rightarrow \angle BDE = 90^\circ - 48^\circ = 42^\circ$   
In  $\triangle BED$   
 $\angle EBD + \angle BDE + \angle BED = 180^\circ$   
(Angle sum property)  
 $\Rightarrow 86^\circ + 42^\circ + \angle BED = 180^\circ$   
 $\Rightarrow \angle BED = 180^\circ - (86^\circ + 42^\circ) = 52^\circ$

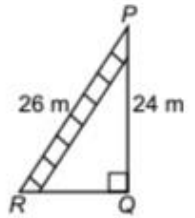
14. (A) : It is given that,  $ABCD$  is a rectangle  
 $\therefore \angle ADC = 90^\circ$   
In  $\triangle ADC$ ,  
 $\angle DAC + \angle ADC + \angle DCA = 180^\circ$   
(Angle sum property)  
 $\Rightarrow 65^\circ + 90^\circ + x = 180^\circ \Rightarrow x = 25^\circ$

15. (D) :  $\angle KLO = \angle MLN$  (Vertically opposite angles)  
 $\therefore \angle MLN = 70^\circ$   
In  $\triangle LMN$ ,  
 $\angle MLN + \angle LNM + \angle LMN = 180^\circ$   
(Angle sum property)

$$\Rightarrow 70^\circ + \angle LNM + 50^\circ = 180^\circ$$

$$\Rightarrow \angle LNM = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$$

16. (A) : In  $\triangle PRQ$ ,  
 $PR^2 = PQ^2 + QR^2$   
(By Pythagoras theorem)  
 $(26)^2 = (24)^2 + QR^2$   
or  $QR^2 = 676 - 576 = 100$   
 $\Rightarrow QR = \sqrt{100} \Rightarrow QR = 10$   
 $\therefore$  The distance of the foot of the ladder from the wall is 10 m.



17. (C) : Let  $KB$  is original height of the tree.  
In  $\triangle ABC$ ,

$$AC^2 = AB^2 + BC^2 = 5^2 + 12^2$$

$$= 25 + 144 = 169$$

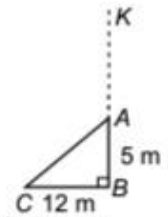
$$\therefore AC = \sqrt{169} = 13 \text{ m}$$

$$KB = KA + AB$$

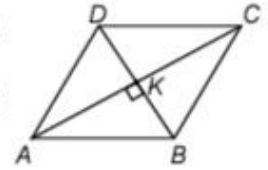
$$= AC + AB \text{ (}\because KA = AC\text{)}$$

$$= (13 + 5) \text{ m} = 18 \text{ m}$$

$$\therefore \text{Original height of the tree is 18 m.}$$



18. (B) : Given :  $BD = 42$  cm  
and  $AC = 56$  cm  
Since diagonals of a rhombus bisect each other at  $90^\circ$ .



$$\therefore BK = \frac{1}{2}BD = \frac{42}{2} \text{ cm} = 21 \text{ cm}$$

$$AK = \frac{1}{2}AC = \frac{56}{2} \text{ cm} = 28 \text{ cm}$$

$$\text{In } \triangle KAB, AB^2 = AK^2 + BK^2$$

$$= (28)^2 + (21)^2 = 784 + 441 = 1225$$

$$\therefore AB = \sqrt{1225} = 35 \text{ cm}$$

$$\therefore \text{Perimeter of the field } ABCD = (4 \times 35) \text{ cm}$$

$$= 140 \text{ cm}$$

19. (B) : Let  $AB$  = length of the ladder,  $AC$  = height of the window

$$\text{In } \triangle ABC,$$

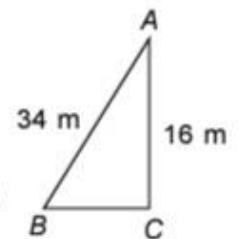
$$(AB)^2 = (AC)^2 + (BC)^2$$

$$\Rightarrow (34)^2 = (16)^2 + BC^2$$

$$\text{or } BC^2 = (34)^2 - (16)^2$$

$$\Rightarrow BC^2 = 1156 - 256 = 900$$

$$\therefore BC = \sqrt{900} = 30 \text{ m}$$



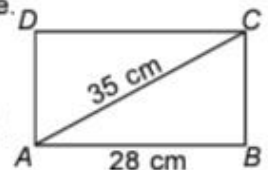
20. (D) :  $ABCD$  is a rectangle.

$$\text{In } \triangle ACB,$$

$$AC^2 = AB^2 + BC^2$$

$$\text{(By Pythagoras theorem)}$$

$$(35)^2 = (28)^2 + BC^2 \text{ or } BC^2$$



$$\begin{aligned}
 &= (35)^2 - (28)^2 \\
 &\Rightarrow BC^2 = 1225 - 784 \Rightarrow BC^2 = 441 \\
 &\therefore BC = \sqrt{441} = 21 \text{ cm} \\
 &\therefore \text{Perimeter of rectangle} = 2 \times (28 + 21) \text{ cm} \\
 &= 2 \times (49) \text{ cm} = 98 \text{ cm}
 \end{aligned}$$

21. (C) : As,  $y = 5x$

$$\begin{aligned}
 &\therefore \text{In } \triangle RQS, \\
 &x + y + 60^\circ = 180^\circ \quad (\text{Angle sum property}) \\
 &\Rightarrow x + 5x + 60^\circ = 180^\circ \\
 &\Rightarrow 6x = 180^\circ - 60^\circ = 120^\circ \Rightarrow x = \frac{120^\circ}{6} = 20^\circ
 \end{aligned}$$

$$\begin{aligned}
 &\therefore y = 5 \times 20^\circ = 100^\circ \\
 &\text{Also } \angle QRS + \angle QSR = z \\
 &\hspace{15em} (\text{Exterior angle property}) \\
 &\Rightarrow z = 60^\circ + 100^\circ = 160^\circ
 \end{aligned}$$

22. (B) : (i) In the given right angled triangle,

$$BC^2 = AB^2 + AC^2$$

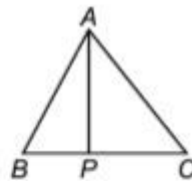
(iii) A triangle can have three medians.

23. (C)

24. (C) : Statement - 2

$$\text{In } \triangle ABP, \\ AB + BP > AP \quad \dots(i)$$

$$\text{In } \triangle APC \\ PC + AC > AP \quad \dots(ii)$$



Adding (i) & (ii), we get

$$AB + BP + PC + AC > AP + AP$$

$$\Rightarrow AB + BC + AC > 2AP$$

$\therefore$  Both Statement -1 and Statement-2 are true.

25. (B) : In  $\triangle TCE$ ,

$$x = \angle TCE + \angle TEC \quad (\text{Exterior angle property})$$

$$\Rightarrow x = 35^\circ + 31^\circ$$

$$\Rightarrow x = 66^\circ$$

In  $\triangle SBD$ ,

$$\angle AST = \angle SBD + \angle SDB \\ \hspace{15em} (\text{Exterior angle property})$$

$$\angle AST = 30^\circ + 36^\circ = 66^\circ$$

In  $\triangle ATS$ ,

$$y + x + \angle AST = 180^\circ \quad (\text{Angle sum property})$$

$$\Rightarrow y + 66^\circ + 66^\circ = 180^\circ$$

$$\Rightarrow y = 180^\circ - (66^\circ + 66^\circ) \Rightarrow y = 48^\circ$$

# SELF ASSESSMENT EXERCISE

## MULTIPLE-CHOICE QUESTIONS (MCQ)

Choose the correct answer in each of the following questions:

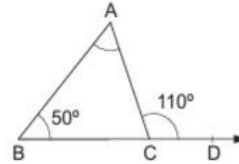
1. In a  $\triangle ABC$ , if  $3\angle A = 4\angle B = 6\angle C$  then  $A : B : C = ?$   
 (a) 3 : 4 : 6      (b) 4 : 3 : 2      (c) 2 : 3 : 4      (d) 6 : 4 : 3

2. In a  $\triangle ABC$ , if  $\angle A - \angle B = 42^\circ$  and  $\angle B - \angle C = 21^\circ$  then  $\angle B = ?$

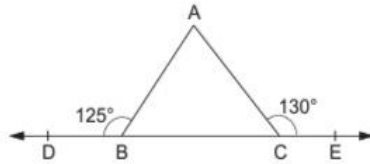
- (a)  $32^\circ$       (b)  $63^\circ$       (c)  $53^\circ$       (d)  $95^\circ$

3. In a  $\triangle ABC$ , side  $BC$  is produced to  $D$ . If  $\angle ABC = 50^\circ$  and  $\angle ACD = 110^\circ$  then  $\angle A = ?$

- (a)  $160^\circ$       (b)  $60^\circ$   
 (c)  $80^\circ$       (d)  $30^\circ$



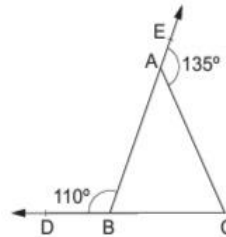
4. Side  $BC$  of  $\triangle ABC$  has been produced to  $D$  on left and to  $E$  on right-hand side of  $BC$  such that  $\angle ABD = 125^\circ$  and  $\angle ACE = 130^\circ$ . Then,  $\angle A = ?$



- (a)  $50^\circ$       (b)  $55^\circ$       (c)  $65^\circ$       (d)  $75^\circ$

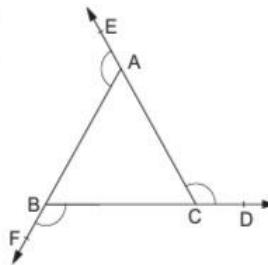
5. In the given figure, the sides  $CB$  and  $BA$  of  $\triangle ABC$  have been produced to  $D$  and  $E$  respectively such that  $\angle ABD = 110^\circ$  and  $\angle CAE = 135^\circ$ . Then,  $\angle ACB = ?$

- (a)  $65^\circ$       (b)  $45^\circ$   
 (c)  $55^\circ$       (d)  $35^\circ$



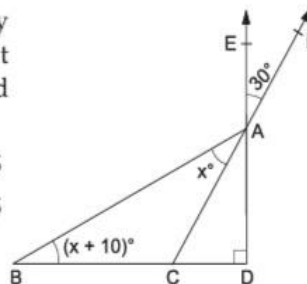
6. The sides  $BC$ ,  $CA$  and  $AB$  of  $\triangle ABC$  have been produced to  $D$ ,  $E$  and  $F$  respectively.  $\angle BAE + \angle CBF + \angle ACD = ?$

- (a)  $240^\circ$       (b)  $300^\circ$   
 (c)  $320^\circ$       (d)  $360^\circ$

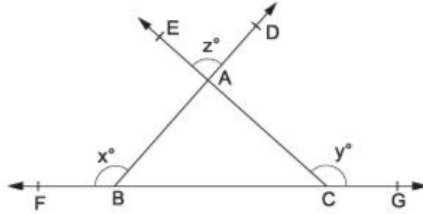


7. In the given figure,  $EAD \perp BCD$ . Ray  $FAC$  cuts ray  $EAD$  at a point  $A$  such that  $\angle EAF = 30^\circ$ . Also, in  $\triangle BAC$ ,  $\angle BAC = x^\circ$  and  $\angle ABC = (x + 10)^\circ$ . Then, the value of  $x$  is

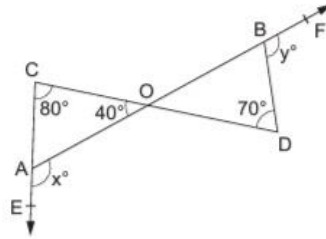
- (a) 20      (b) 25  
 (c) 30      (d) 35



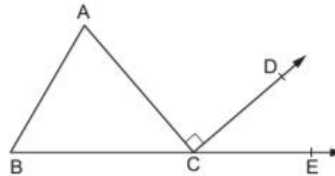
8. In the given figure, two rays  $BD$  and  $CE$  intersect at a point  $A$ . The side  $BC$  of  $\triangle ABC$  have been produced on both sides to points  $F$  and  $G$  respectively. If  $\angle ABF = x^\circ$ ,  $\angle ACG = y^\circ$  and  $\angle DAE = z^\circ$  then  $z = ?$



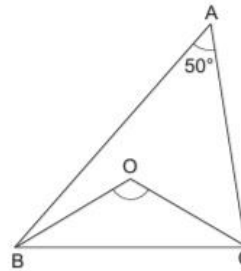
- (a)  $x + y - 180$  (b)  $x + y + 180$  (c)  $180 - (x + y)$  (d)  $x + y + 360^\circ$
9. In the given figure, lines  $AB$  and  $CD$  intersect at a point  $O$ . The sides  $CA$  and  $OB$  have been produced to  $E$  and  $F$  respectively such that  $\angle OAE = x^\circ$  and  $\angle DBF = y^\circ$ .



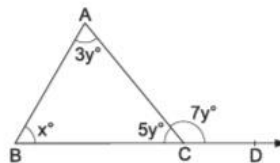
- If  $\angle OCA = 80^\circ$ ,  $\angle COA = 40^\circ$  and  $\angle BDO = 70^\circ$  then  $x^\circ + y^\circ = ?$
- (a)  $190^\circ$  (b)  $230^\circ$  (c)  $210^\circ$  (d)  $270^\circ$
10. In a  $\triangle ABC$ , it is given that  $\angle A : \angle B : \angle C = 3 : 2 : 1$  and  $\angle ACD = 90^\circ$ . If  $BC$  is produced to  $E$  then  $\angle ECD = ?$
- (a)  $60^\circ$   
 (b)  $50^\circ$   
 (c)  $40^\circ$   
 (d)  $25^\circ$



11. In the given figure,  $BO$  and  $CO$  are the bisectors of  $\angle B$  and  $\angle C$  respectively. If  $\angle A = 50^\circ$  then  $\angle BOC = ?$
- (a)  $130^\circ$  (b)  $100^\circ$   
 (c)  $115^\circ$  (d)  $120^\circ$



12. In the given figure, side  $BC$  of  $\triangle ABC$  has been produced to a point  $D$ . If  $\angle A = 3y^\circ$ ,  $\angle B = x^\circ$ ,  $\angle C = 5y^\circ$  and  $\angle CBD = 7y^\circ$ . Then, the value of  $x$  is
- (a) 60 (b) 50  
 (c) 45 (d) 35



**ANSWERS (MCQ)**

1. (b) 2. (c) 3. (b) 4. (d) 5. (a) 6. (d) 7. (b) 8. (a)  
 9. (b) 10. (a) 11. (c) 12. (a)

