ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-1

SUBJECT – MATHEMATICS

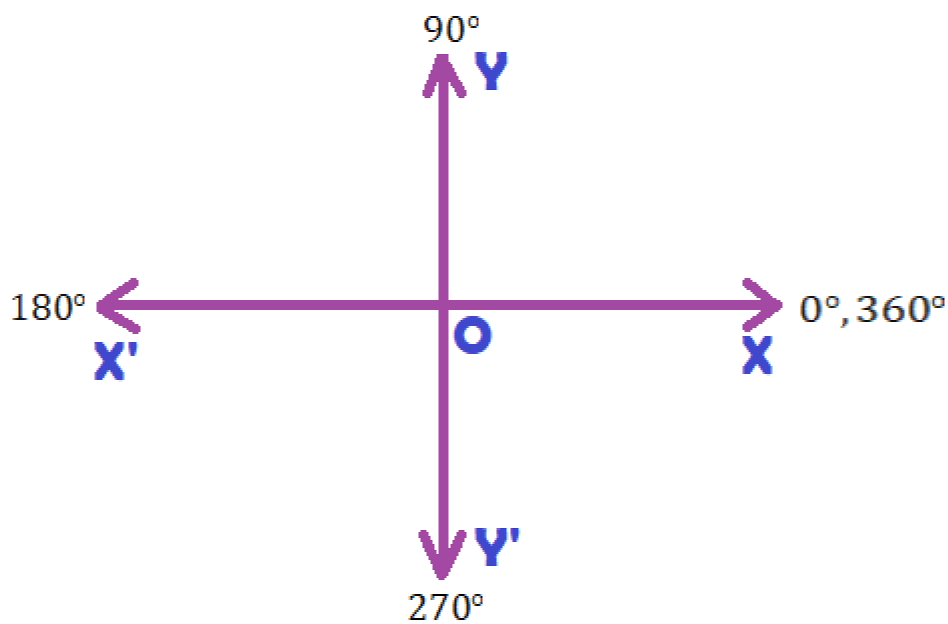
1st term

Chapter: Trigonometry **Class: XI**

Topic: Associated angles **Date: 17.06.2020**

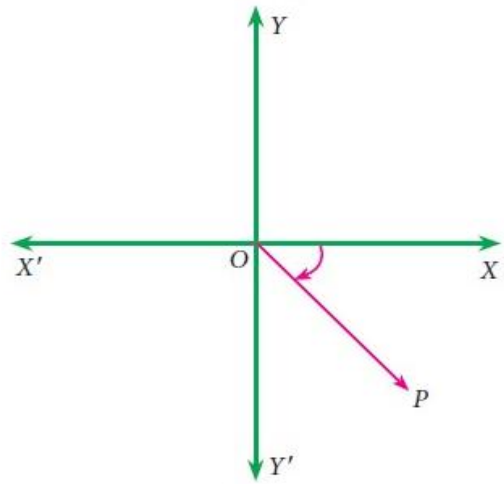
1. **How to determine the position of the terminal side of an angle in our co-ordinate system :-**

Take OX as initial side of an angle. Then rotate it accordingly . By rotating it anti-clock wise, we get a positive angle and by rotating it clock wise, we get a negative angle.

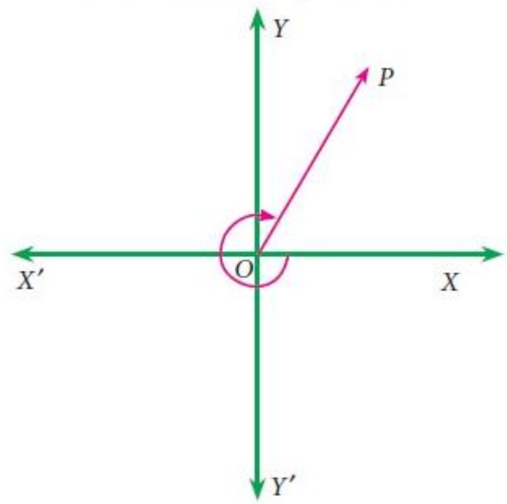


➢ For example –

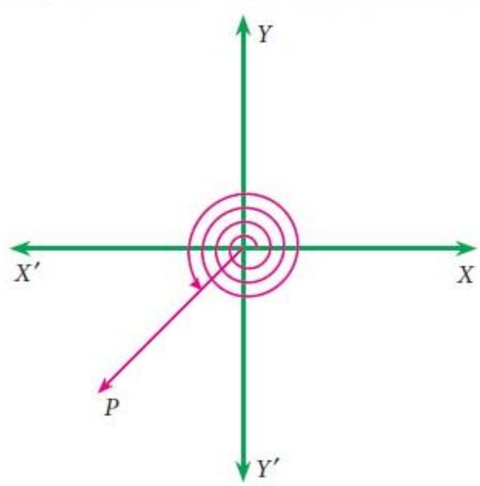
(i) The terminal side of -70° lies in IV quadrant.



(ii) The terminal side -320° of lies in I quadrant.

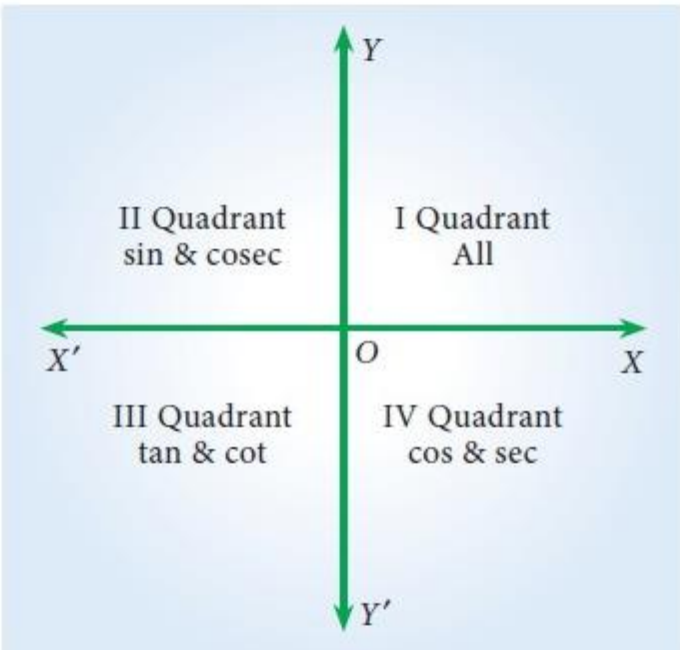


(iii) $1325^\circ = (3 \times 360) + 180^\circ + 65^\circ$ the terminal side lies in III quadrant.



2. Signs of the trigonometric ratios of an angle θ as it varies from 0° to 360°

In the first quadrant both x and y are positive. So all trigonometric ratios are positive. In the second quadrant ($90° < \theta < 180°$) x is negative and y is positive. So trigonometric ratios sin θ and cosec θ are positive. In the third quadrant ($180° < \theta < 270°$) both x and y are negative. So trigonometric ratios tan θ and cot θ are positive. In the fourth quadrant ($270° < \theta < 360°$) x is positive and y is negative. So trigonometric ratios cos θ and sec θ are positive.



ASTC : All Sin Tan Cos

3. Trigonometric ratios of angles associated with a given angle θ.

Angle/ Function	$-\theta$	$90°-\theta$ or $\frac{\pi}{2}-\theta$	$90°+\theta$ or $\frac{\pi}{2}+\theta$	$180°-\theta$ or $\pi-\theta$	$180°+\theta$ or $\pi+\theta$	$270°-\theta$ or $\frac{3\pi}{2}-\theta$	$270°+\theta$ or $\frac{3\pi}{2}+\theta$	$360°-\theta$ or $2\pi-\theta$	$360°+\theta$ or $2\pi+\theta$
Sine	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
Cosine	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
tangent	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$
cotangent	$-\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$
secant	$\sec\theta$	$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\,\theta$	$\text{cosec}\,\theta$	$\sec\theta$	$\sec\theta$
cosecant	$-\text{cosec}\,\theta$	$\sec\theta$	$\sec\theta$	$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\,\theta$	$\text{cosec}\,\theta$

Note:

1. The relations of all the trigonometrical ratios of a positive acute angle and angle θ can either be positive or negative.

2. The values of sin θ cannot be greater than 1 i.e., $-1 \le \sin\theta \le 1$

3. The values of cos θ cannot be greater than 1 i.e., $-1 \le \cos\theta \le 1$

4. The values of sec θ cannot be less than 1 i.e., $\sec\theta \ge 1$ or $\sec\theta \le -1$

5. The values of csc θ cannot be less than 1 i.e., $\csc\theta \ge 1$ or $\csc\theta \le -1$

6. Tan θ and cot θ can have any real values.

Read $\csc\theta$ *as* $\operatorname{cosec}\theta$.

Example 1.

Find the values of each of the following trigonometric ratios.

(i) $\sin 150^\circ$ (ii) $\cos(-210^\circ)$ (iii) $\operatorname{cosec} 390^\circ$ (iv) $\tan(-1215^\circ)$

(v) $\sec 1485^\circ$

Solution

(i) $\sin 150^\circ = \sin(1\times 90^\circ + 60^\circ)$

Since 150° lies in the second quadrant, we have

$$\sin 150^\circ = \sin(1\times 90^\circ + 60^\circ)$$

$$= \cos 60^\circ = \frac{1}{2}$$

(ii) we have $\cos(-210^\circ) = \cos 210^\circ$

Since 210° lies in the third quadrant, we have

$$\cos 210^\circ = \cos(180^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

(iii) $\operatorname{cosec} 390^\circ = \operatorname{cosec}(360^\circ + 30^\circ) = \operatorname{cosec} 30^\circ = 2$

(iv) $\tan(-1215^\circ) = -\tan(1215^\circ) = -\tan(3\times 360^\circ + 135^\circ)$

$$= -\tan 135^\circ = -\tan(90^\circ + 45^\circ) = -(-\cot 45^\circ) = 1$$

(v) $\sec 1485^\circ = \sec(4\times 360^\circ + 45^\circ) = \sec 45^\circ = \sqrt{2}$

Example 2.

Prove that $\sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ = -1$

Solution

$$\sin 600^\circ = \sin(360^\circ + 240^\circ) = \sin 240^\circ = \sin(180^\circ + 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

$$\cos 390^\circ = \cos(360^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 480^\circ = \cos(360^\circ + 120) = \cos 120^\circ$$

$$= \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$$

$$\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

Now $\sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{3}{4} - \frac{1}{4} = \frac{-3-1}{4} = -1$$

HOME-WORK

1. Convert the following degree measure into radian measure

   (i) 60° (ii) 150° (iii) 240° (iv) -320°

2. Find the degree measure corresponding to the following radian measure.

   (i) $\frac{\pi}{8}$ (ii) $\frac{9\pi}{5}$ (iii) -3 (iv) $\frac{11\pi}{18}$

3. Determine the quadrants in which the following degree lie.

   (i) 380° (ii) -140° (iii) 1195°

4. Find the values of each of the following trigonometric ratios.

   (i) $\sin 300^\circ$ (ii) $\cos(-210^\circ)$ (iii) $\sec 390^\circ$ (iv) $\tan(-855^\circ)$

   (v) $\text{cosec} 1125^\circ$

5. Prove that: (i) $\tan(-225^\circ)\cot(-405^\circ) - \tan(-765^\circ)\cot(675^\circ) = 0$

   (ii) $2\sin^2\frac{\pi}{6} + \text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

   (iii) $\sec\left(\frac{3\pi}{2} - \theta\right)\sec\left(\theta - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + \theta\right)\tan\left(\theta - \frac{5\pi}{2}\right) = -1$

6. If A, B, C, D are angles of a cyclic quadrilateral, prove that :

$\cos A + \cos B + \cos C + \cos D = 0$

7. Prove that : (i) $\dfrac{\sin(180^\circ-\theta)\cos(90^\circ+\theta)\tan(270-\theta)\cot(360-\theta)}{\sin(360-\theta)\cos(360+\theta)\sin(270-\theta)\operatorname{cosec}(-\theta)} = -1$

(ii) $\sin\theta\cdot\cos\theta\left\{\sin\left(\frac{\pi}{2}-\theta\right)\cdot\operatorname{cosec}\theta+\cos\left(\frac{\pi}{2}-\theta\right)\cdot\sec\theta\right\} = 1$

8. Prove that : $\cos 510^\circ \cos 330^\circ + \sin 390^\circ \cos 120^\circ = -1$

9. Prove that: (i) $\tan(\pi+x)\cot(x-\pi)-\cos(2\pi-x)\cos(2\pi+x) = \sin^2 x$

(ii) $\dfrac{\sin(180^\circ+A)\cos(90^\circ-A)\tan(270^\circ-A)}{\sin(540^\circ-A)\cos(360^\circ+A)\operatorname{cosec}(270^\circ+A)} = -\sin A\cos^2 A$

10. If $\sin\theta = \frac{3}{5}$, $\tan\varphi = \frac{1}{2}$ and $\frac{\pi}{2} < \theta < \pi < \varphi < \frac{3\pi}{2}$, then find the value of $8\tan\theta - \sqrt{5}\sec\varphi$

Prepared by -

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