ST. LAWRENCE HIGH SCHOOL
A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-14

SUBJECT – STATISTICS

Pre-test

Chapter: THEORITICAL PROBABILITY DISTRIBUTION **Class: XII**

Topic: POISSON PROBABILITY DISTRIBUTION **Date: 26.06.20**

PROBABILITY DISTRIBUTION

PART 8

A random variable X follows Poisson distribution with parameter λ

$$X \sim \text{poissn}(\lambda)$$

The pmf of the random variable X is given by

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, x = 0(1)\infty$$

PROPERTIES:

4. **Recursion relation between the central moments**

solution:

$$f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$\Rightarrow \frac{d}{d\lambda}\frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{1}{x!}e^{-\lambda}(x\lambda^{x-1} - \lambda^{x})$$

$$\Rightarrow \frac{d}{d\lambda}\frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{1}{x!}e^{-\lambda}\lambda^{x}\left(\frac{x}{\lambda} - 1\right)$$

$$\Rightarrow \frac{d}{d\lambda}\frac{e^{-\lambda}\lambda^{x}}{x!} = f(x)\left(\frac{x}{\lambda} - 1\right)$$

$$\Rightarrow \frac{d}{d\lambda}\frac{e^{-\lambda}\lambda^{x}}{x!} = f(x)\left(\frac{x-\lambda}{\lambda}\right)$$

The rth order central moment

$$\mu_r = E(X-\lambda)^r$$

$$\frac{d}{d\lambda}\mu_r = \frac{d}{d\lambda}\sum_{x=0}^{\infty} f(x)\,(x-\lambda)^r$$

$$\Rightarrow \frac{d}{d\lambda}\mu_r = \sum_{x=0}^{\infty} -r\,(x-\lambda)^{r-1} f(x) + \frac{1}{\lambda}\sum_{x=0}^{\infty}(x-\lambda)^{r+1} f(x)$$

$$\Rightarrow \frac{d}{d\lambda}\mu_r = -r\,\mu_{r-1} + \frac{1}{\lambda}\,\mu_{r+1}$$

Taking r = 2,

$$\frac{d}{d\lambda}\mu_2 = -r\,\mu_1 + \frac{1}{\lambda}\,\mu_3$$

$$\Rightarrow \mu_3 = \lambda$$

So $\gamma_1 = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{1}{\sqrt{\lambda}} > 0$ which implies Poisson distribution is positively skewed distribution.

5. Mean deviation about mean

$$E(|X-\lambda|)$$

$$= \sum_{x=k+1}^{\infty}(x-\lambda)f(x) \qquad where\ k = [\lambda]$$

$$= \sum_{x=k+1}^{\infty}(x-\lambda)\frac{e^{-\lambda}\,\lambda^{x}}{x!}$$

$$= \sum_{x=k+1}^{\infty}\left\{\frac{e^{-\lambda}\,\lambda^{x}}{(x-1)!} - \frac{e^{-\lambda}\,\lambda^{x+1}}{x!}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$Define\ \gamma_x = \frac{e^{-\lambda}\lambda^x}{(x-1)!} \Rightarrow \gamma_{x+1} = \frac{e^{-\lambda}\lambda^{x+1}}{x!}$

From (1), $\sum_{x=k+1}^{\infty}(\gamma_x - \gamma_{x+1}) = \gamma_{k+1}$

$$= \frac{e^{-\lambda}\lambda^{k+1}}{k!}$$

6. Mode of Poission distribution

Using the relation $f(x) \geq f(x+1)$

$$\Rightarrow \frac{e^{-\lambda}\lambda^x}{x!} \geq \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$$

$$\Rightarrow x+1 \geq \lambda$$

$$\Rightarrow x \geq \lambda - 1$$

Using the relation $f(x) \geq f(x-1)$

$$\Rightarrow \frac{e^{-\lambda}\lambda^x}{x!} \geq \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}$$

$$\Rightarrow x \leq \lambda$$

Case 1: When the distribution is unimodal and mode at X=k

$f(0) < f(1) < \ldots\ldots < f(k-1) < f(k) > f(k+1) > f(k+2) > \ldots.$

Here mode is at X = [λ]

Case 2: When the distribution is bimodal and mode at X=k-1 and X=k

$$f(0) < f(1) < \ldots \ldots < f(k-1) = f(k) > f(k+1) > f(k+2) > \ldots.$$

Here mode is at X = $\lambda - 1$ and X = λ.

Prepared by

Sanjay Bhattacharya