ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-7

SUBJECT – STATISTICS

Pre-test

Chapter: BINOMIAL PROBABILITY DISTRIBUTION **Class: XII**

Topic:CORRELATION FOR GROUPED DATA **Date: 18.06.20**

PROBABILITY DISTRIBUTION

PART 1

A discreterandom variable X follows Binomial distribution with parameters

n and p,

ie, X$\sim Bin(n, p\ )$

The p.m.f of the variable is given by $f(x) = \ n_{C_x}\ p^x\ (1-p)^{n-x}\ \ for\ all\ x = 0(1)n$

Where n = number of trials

P= probability of success in a single trial

X= required number of success.

To check that it is a pmf.

$n_{C_x} > 0, p\ \geq 0\ and\ (1-p) \geq 0, hence\ f(x) \geq 0.$

$$\sum_{i=0}^{n} f(x) = (\ 1 - p + p\ )^n\ = 1. \Rightarrow \sum_{i=0}^{n} f(x) = 1$$

Hence f(x) is a pmf.

Condition for Binomial distribution:

Binomial distribution is used in case of Bernoullian variable, ie,

(i) Number of trials are countably finite
(ii) In each trials there exists only two possible outcomes, viz, success and failure
(iii) Probability of success in every single trial is same.

Eg, An unbiased die is rolled five times. Find the probability of getting three 6's.

Soln: X:number of 6's appear in five throws of an unbiased die.

Therefore, X$\sim Bin(n, p\ )$ where $\mathrm{n} = 5$ and $\mathrm{p} = \frac{1}{6}$

So P(X=3) =$f(3) = \ 5_{C_3}\ (\frac{1}{6})^3\ \left(1-\ \frac{1}{6}\right)^{5-3}\ = \frac{250}{6^5}$

Properties of Bin(n, p)

$$\sum_{x=0}^{n} x(x-1)(x-2)\ \ldots\ \ldots \ldots (x-r+1) f(x)$$

$$= \ \sum_{x=0}^{n} x(x-1)(x-2)\ \ldots\ \ldots \ldots (x-r+1)\ \frac{n!}{(n-x)!x!}\ p^x\ \ (1-p)^{n-x}$$

$$= \ \sum_{x=0}^{n} n(n-1)(n-2)\ \ldots\ \ldots \ldots (n-r+1)\ \frac{n!}{(n-x)!x!}\ p^x\ (1-p)^{n-x}$$

$$= n(n-1)(n-2)..(n-r+1)\ \sum_{x-r=0}^{n-r} \frac{(n-r)!}{\left((n-r)-(x-r)\right)!(x-r)!}\ p^x\ (1-p)^{n-x}$$

$$= \ n(n-1)(n-2)..(n-r+1)\ p^r\ (1-p+p)^{n-r}$$

$$= \ n(n-1)(n-2)..(n-r+1)\ p^r$$

X$\boldsymbol{\sim Bin(n, p\ )}$

1. **r th order factorial momement of the random variable X**

$$\mu_{[r]} = \mathrm{E(X(X-1)(x-2)\ \ldots \ldots \ldots \ldots \qquad (X-r+1))}$$
$$= \ \sum_{i=0}^{n} x(x-1)(x-2)\ \ldots\ \ldots \ldots (x-r+1) f(x)$$
$$= n\ (n-1)(n-2)..(n-r+1)\ p^r$$

2. Expectation and variance of the random variable X

Put r= 1, E(X) = np

$r = 2, \quad E(x(x-1) = \ n(n-1)p^2$

Variance of X = $\sigma_X^2 = E\big(X(X-1)\big) + \ E(X) - (E(X))^2$

$$= n(n-1)p^2 + \ np - \ n^2p^2$$

$$= np(1-p)$$

3. Binomial distribution is symmetric when $\boldsymbol{p} = \frac{1}{2}$

A real valued mathematical function with domain [a, b] is symmetric iff

f(a + x) =f(b – x)

Using the property for binomial pmf.

f (0+ x) = f (n – x)

$\Rightarrow \ \ n_{C_x} \ p^x \ (1-p)^{n-x} \ \ = \ \ n_{C_{n-x}} \ p^{n-x} \ (1-p)^x$

$\Rightarrow \ \ (1-p)^{n-2x} \ \ = \ \ \ p^{n-2x}$

$\Rightarrow 1 - p = p$

$\Rightarrow p = \dfrac{1}{2}$

So Binomial dic when istribution is symmetric when $p = \frac{1}{2}$.

4. Binomial distribution attains maximum variance when $p = \frac{1}{2}$

$$\sigma_X^2 = np(1-p)$$

$$\Rightarrow \frac{d}{dp}\sigma_X^2 = n - 2np$$

$$\Rightarrow \frac{d^2}{dp^2}\sigma_X^2 = -2n < 0 \Rightarrow always\ maximum$$

$$\frac{d}{dp}\sigma_X^2 = 0 \Rightarrow n - 2np = 0 \Rightarrow p = \frac{1}{2}$$

So Binomial distribution attains maximum variance when $p = \frac{1}{2}$

Hence symmetric Binomial distribution has maximum variance.

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