ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-17

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Integration **Class: XII**

Topic: Method Of Substitution **Date: 26.06.2020**

–:Method of Substitution:–

1 $I=\int f(\phi(x))\phi'(x)\,dx$: Here, we put $\phi(x)=t$, so that $\phi'(x)\,dx=dt$ and in that case

$$\int f(\phi(x))\phi'(x)\,dx=\int f(t)\,dt$$

Example 1. Evaluate $\int x^3\sin x^4\,dx$.

Solution: We have

$$I=\int x^3\sin x^4\,dx$$

Let $x^4=t$. Then

$$4x^3\,dx=dt\Rightarrow dx=\frac{dt}{4x^3}$$

$$\Rightarrow I=\int\frac{\sin t}{4}dt=-\frac{\cos t}{4}+c=\frac{\cos x^4}{4}+c$$

Example 2. Evaluate $\int\frac{\sin(\ln x)}{x}dx$.

Solution: We have

$$I=\int\frac{\sin(\ln x)}{x}dx$$

Let $\ln x=t$. Then

$$\frac{dx}{x}=dt$$

$$\Rightarrow I=\int\sin t\,dt=-\cos t+c-\cos(\ln x)+c$$

Example 3. Evaluate $\int \frac{x}{x^4+x^2+1}dx.$

Solution: We have

$$I=\int \frac{x}{x^4+x^2+1}dx$$

Let $x^2=t$. Then $2xdx=dt$.

$$I=\int \frac{x}{x^4+x^2+1}dx=\frac{1}{2}\int \frac{1}{t^2+t+1}dt$$

$$I=\frac{1}{2}\int \frac{1}{\left(t+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dt$$

$$\Rightarrow I=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\left(t+\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\right)+c=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\left(x^2+\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\right)+c$$

2 $I=\int f(x)\cdot f'(x)dx$: In this case, we put $f(x)=t \Rightarrow f'(x)dx=dt$

Example 4. Evaluate $\int \sin x\cdot\cos x\,dx$.

Solution: We have

$$I=\int \sin x\cdot\cos x\,dx$$

Let $\sin x=t$. Then $\cos x dx=dt$.

$$I=\int \sin x\cdot\cos x\,dx=\int t\,dt$$

$$\Rightarrow I=\frac{t^2}{2}+c=\frac{(\sin x)^2}{2}+c$$

Example 5. Evaluate $\int \frac{x^2\tan^{-1}x^3}{1+x^6}dx$.

Solution:

$$I=\int \frac{x^2\tan^{-1}x^3}{1+x^6}dx$$

Put $\tan^{-1}x^3=t$. Then

$$\frac{3x^2}{1+x^6}dx=dt$$

$$\Rightarrow I=\frac{1}{3}\int tdt=\frac{1}{3}\cdot\frac{t^2}{2}+c=\frac{(\tan^{-1}x^3)^2}{6}+c$$

3 $I=\int \frac{f'(x)}{f(x)}dx$: In this case, we put $f(x)=t$ and $f'(x)dx=dt$. So,

$$I=\int \frac{f'(x)}{f(x)}\,dx=\int \frac{1}{t}\,dt=\ln(f(x))+c$$

Example 6. Evaluate $\int \frac{x^3}{1+x^4}dx.$

Solution:

$$I=\int \frac{x^3}{1+x^4}dx$$

Put $1+x^4=t$. Then $\Rightarrow 4x^3dx=dt$

$$\Rightarrow I=\frac{1}{4}\int \frac{1}{t}dt=\frac{1}{4}\ln t+c=\frac{1}{4}\ln(1+x^4)+c$$

Example 7. Evaluate $\int \frac{1}{1+e^x}dx.$

Solution:

$$I=\int \frac{1}{1+e^x}dx=\int \frac{e^{-x}}{1+e^{-x}}dx$$

Put $1+e^{-x}=t$. Then $-e^{-x}dx=dt$

$$\Rightarrow I=-\int \frac{1}{t}dt=-\ln t+c=-\ln(1+e^{-x})+c$$

Example 8. Evaluate $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx.$

Solution:

$$I = \int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Put $a^2 \sin^2 x + b^2 \cos^2 x = t$. Then $\sin 2x (a^2 - b^2) dx = dt$

$$\Rightarrow I = \frac{1}{(a^2 - b^2)} \int \frac{1}{t} dt = \frac{1}{(a^2 - b^2)} \ln t + c$$

$$= \frac{1}{(a^2 - b^2)} \ln(a^2 \sin^2 x + b^2 \cos^2 x) + c$$

4 $I = \int (f(x))^n \cdot f'(x) dx$: In this case, we put $f(x) = t$ and $f'(x) dx = dt$. So,

$$I = \int (f(x))^n \cdot f'(x) dx = \frac{(f(x))^{n+1}}{n+1} + c$$

Example 9. Evaluate $\int \frac{(\ln x)^5}{x}\,dx.$

Solution:

$$I = \int \frac{(\ln x)^5}{x}\,dx$$

Put $\ln x = t$. Then $\frac{dx}{x} = dt$

$$\Rightarrow I = \int t^5 dt = \frac{t^6}{6} + c = \frac{(\ln x)^6}{6} + c$$

Example 10. Evaluate $\int \sin^{10} x \cdot \cos x\,dx.$

Solution:

$$I = \int \sin^{10} x \cdot \cos x\,dx$$

Put $\sin x = t$. Then $\cos x\,dx = dt$

$$\Rightarrow I = \int t^{10} dt = \frac{t^{11}}{11} + c = \frac{(\sin x)^{11}}{11} + c$$

Example 11. Evaluate $\int \cos 3x \cdot \sqrt{2+\sin 3x}\, dx$.

Solution:

$$I = \int \cos 3x \cdot \sqrt{2+\sin 3x}\, dx$$

Put $2+\sin 3x = t$. Then

$$3\cos 3x\, dx = dt$$

$$\Rightarrow I = \frac{1}{3}\int t^{\frac{1}{2}} dt = \frac{1}{3}\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{2(2+\sin 3x)^{\frac{3}{2}}}{9} + c$$

- **Prepared by**

Mr. Sukumar Mandal (SkM)