ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-3

SUBJECT – STATISTICS

1st term

Chapter: INTERPOLATION **Class: XI**

Topic: Newtons forward formula **Date: 25.06.20**

INTERPOLATION

PART 1

Given some values of arguments and entries the technique by which we can find the value of an entry for any intermediate value of the argument is known as interpolation.

The arguments are known as the independent variables and entries dependent variables.

Consider x: arguments and y: entry and y=f(x).

Given n+1 pairs of values of arguments and corresponding entries the polynomial can be formed, is of degree n.

Assumption: all the arguments are equidistant, i.e, those are in A.P.

Let us consider h= the common difference or the common interval between the successive arguments.

Operators used in interpolation:

Δ**Operator**:

$\Delta f(x) = f(x+h) - f(x)$

So $\Delta f(x_r) = f(x_r + h) - f(x_r)$

$\Rightarrow \Delta f(x_r) = f(x_{r+1}) - f(x_r)$

$\Rightarrow \Delta y_r = y_{r+1} - y_r$

$\Rightarrow \Delta y_0 = y_1 - y_0$

$$\Delta^2 f(x) = \Delta(\Delta f(x))$$

$$= \Delta\,(f(x+h) - f(x))$$

$$= \Delta f(x+h) - \Delta f(x)$$

$$= f(x+2h) - 2\,f(x+h) + f(x)$$

So $\Delta^2 f(x_r) = f(x_r + 2h) - 2\,f(x_r + h) + f(x_r)$

$\Rightarrow \;\; \Delta^2 f(x_r) = f(x_{r+2}) - 2\,f(x_{r+1}) + f(x_r)$

$\Rightarrow \;\; \Delta^2 (y_r) = y_{r+2} - 2\,y_{r+1} + y_r$

$\Rightarrow \;\; \Delta^2 (y_0) = y_2 - 2\,y_1 + y_0$

$$\Delta^3 f(x) = \Delta(\Delta^2 f(x))$$

$$= \Delta(\; f(x+2h) - 2\,f(x+h) + f(x))$$

$$= \Delta f(x+2h) - 2\;\Delta f(x+h) + \;\Delta f(x))$$

$$= f(x+3h) - 3\,f(x+2h) + 3\,f(x+h) - \;f(x)$$

So $\Delta^3 f(x_r) = f(x_r + 3h) - 3\,f(x_r + 2h) + 3f(x_r + h) - \;f(\,x_r)$

$\Rightarrow \;\; \Delta^3 f(x_r) = f(x_{r+3}) - 3\,f(x_{r+2}) + 3\,f(x_{r+1}) - \;f(\,x_r)$

$\Rightarrow \;\; \Delta^3 (y_r) = y_{r+3} - 3\,y_{r+2} + 3y_{r+1} - \;y_r$

$\Rightarrow \;\; \Delta^3 (y_0) = y_3 - 3\,y_2 + 3\,y_1 - y_0$

E operator:

$Ef(x) = f(x+h)$

So $\Delta f(x) = E(f(x)) - f(x)$

$\Rightarrow (\Delta + 1)f(x) = E(f(x))$

$\Rightarrow \Delta + 1 \equiv E$

NEWTON'S FORWARD INTERPOLATION FORM

Given, Argument(x): $x_0 \ \ x_1 \ \ x_2 \ \ .... \ \ x_n$

Entry (y): $y_0 \ \ y_1 \ \ y_2 \ \ .... \ \ y_n$

h : the common difference of arguments and $y = f(x)$

Let the polynomial be

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2) + \cdots + a_n(x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

Put $x = x_0$, $f(x_0) = a_0 \Rightarrow y_0 = a_0$

Put $x = x_1$, $f(x_1) = a_0 + a_1(x_1 - x_0) \Rightarrow y_1 = y_0 + a_1.h$

$$\Rightarrow a_1 = \frac{y_1 - y_0}{h} = \frac{\Delta y_0}{h}$$

Put $x = x_2$, $f(x_2) = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$

$$\Rightarrow y_2 = y_0 + \frac{y_1 - y_0}{h}.2h + a_2 2h.h$$

$\Rightarrow\ y_2 = y_0 +\ 2\,(y_1 - y_0) + a_2 2h.h$

$\Rightarrow\ a_2 = \frac{y_2 - 2y_1 + y_0}{2!\,h^2} = \frac{\Delta^2 y_0}{2!h^2}$

Similarly

$a_r = \frac{\Delta^r y_0}{r!h^r}$ r=1(1) n

Substituting the values of a_r in the equation

$$f(x) = y_0 + \frac{\Delta y_0}{h}(x - x_0) + \frac{\Delta^2 y_0}{2!\,h^2}(x - x_0)(x - x_1) + \frac{\Delta^3 y_0}{3!\,h^3}(x - x_0)(x - x_1)(x - x_2) + \cdots + \frac{\Delta^n y_0}{n!h^n}(x - x_0)(x - x_1)\ldots(x - x_{n-1}) \quad \ldots\ldots\ldots\ldots(2)$$

Take $u = \frac{x - x_0}{h}$

Then $u - k = \frac{x - x_0}{h} - k = \frac{x - x_0 - kh}{h} = \frac{x - x_0 - (x_k - x_0)}{h} = \frac{x - x_k}{h}$

So from (2),

$$f(x) = y_0 +\ u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots + \frac{u(u-1)\ldots(u-n+1)}{n!}\Delta^n y_0$$

Which is known as Newton' forward interpolation formula.

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