ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-5

SUBJECT – STATISTICS

1st term

Chapter: CENTRAL TENDENCY **Class: XI**

Topic: ARITHMETIC MEAN **Date: 30.06.20**

CENTRAL TENDENCY

PART 1

Definition :

The sum of all the observations with degree as the reciprocal of number of observations.

It is denoted by $\bar{x}$.

Case 1 : Ungrouped or raw data

Observations: $x_1, x_2, x_3, \dots \dots \dots, x_n$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Case 2 : Grouped data

Observations: $\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \dots \dots \dots, \mathrm{x}_\mathrm{n}$

Frequencies: $f_1, f_2, f_3, \dots \dots \dots., f_n$

$$\bar{\mathrm{x}} = \frac{1}{\mathrm{N}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_\mathrm{i}\, \mathrm{f}_\mathrm{i} \quad \text{where } \mathrm{N} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f}_\mathrm{i}$$

PROPERTIES:

1. Change of origin or base and scale

If $y_i = a + b\, x_{i\,,} \quad \forall\, i = 1(1)n$

Then $\bar{y} = a + b\bar{x}$

Proof: By definition,

For ungrouped data

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$= \frac{1}{n}\sum_{i=1}^{n}(a + b\, x_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} a + \frac{1}{n}\sum_{i=1}^{n} b x_i$$

$$= \frac{1}{n}\, na + b\bar{x}$$

$$= a + b\bar{x}$$

For grouped data

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{n} y_i\, f_i$$

$$= \frac{1}{N}\sum_{i=1}^{n}(a + b\, x_i) f_i$$

$$= \frac{1}{N}\sum_{i=1}^{n} af_i + \frac{1}{N}\sum_{i=1}^{n} bx_i f_i$$

$$= \frac{1}{N}\ Na + b\bar{x}$$

$$= a + b\bar{x}$$

2. If all the observations are equal to a constant then the am is equal to the same constant.

If $x_i\ = k, \forall\ i = 1(1)n$

Proof:

For ungrouped data

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$= \frac{1}{n}\sum_{i=1}^{n} k$$

$$= \frac{1}{n}\ nk$$

$$= k$$

For grouped data

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i\, f_i$$

$$= \frac{1}{N}\sum_{i=1}^{n} kf_i$$

$= \frac{1}{N}\ kN$= k

3. AM.of all the observations lies between the minimum and maximum observations.

Proof:

Observation: $x_1, x_2, x_3, \ldots \ldots \ldots, x_n$

To show $x_{(1)} \leq \bar{x} \leq x_{(n)}$

$x_{(1)} = \min\{\ x_1, x_2, x_3, \ldots \ldots \ldots, x_n\}$ and

$x_{(n)} = max\ x_1, x_2, x_3, \ldots \ldots \ldots, x_n\}$

For ungrouped data

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ \leq \frac{1}{n}\sum_{i=1}^{n} x_{(n)} = \ x_{(n)}$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \geq \frac{1}{n}\sum_{i=1}^{n} x_{(1)} = x_{(1)}$$

For grouped data

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i f_i \leq \frac{1}{N}\sum_{i=1}^{n} x_{(n)} f_i = \frac{1}{N}.N.x_{(n)} = x_{(n)}$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i f_i \geq \frac{1}{N}\sum_{i=1}^{n} x_{(1)} f_i = \frac{1}{N}.N.x_{(1)} = x_{(1)}$$

Combining we can say,

$$x_{(1)} \leq \bar{x} \leq x_{(n)}$$

4. Combined or composite arithmetic mean

Set 1: observation: $x_{11}, x_{12}, x_{13}, \ldots \ldots \ldots, x_{1n_1}$ with am= $\overline{x_1}$

Set 2: observation: $x_{21}, x_{22}, x_{23}, \ldots \ldots \ldots, x_{2n_2}$ with am= $\overline{x_2}$

Then the combined AM, $\bar{X} = \frac{n_1\overline{x_1}+n_2\overline{x_2}}{n_1+n_2}$

Proof:

By definition,

$$\bar{X} = \frac{\frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i} + \frac{1}{n_2}\sum_{i=1}^{n_2} x_{2i}}{n_1 + n_2}$$

$$\bar{X} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2}$$

Since, $\overline{x_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i}$ and $\overline{x_2} = \frac{1}{n_2}\sum_{i=1}^{n_2} x_{2i}$

5. Then the combined AM, $\bar{X}$ lies between $\overline{x_1}\ and\ \overline{x_2}$

Proof:

Without loss of generality take c

$$\bar{X} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2} < \frac{n_1\overline{x_2} + n_2\overline{x_2}}{n_1 + n_2} = \overline{x_2}$$

$$\bar{X} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2} > \frac{n_1\overline{x_1} + n_2\overline{x_1}}{n_1 + n_2} = \overline{x_1}$$

Hence $\overline{x_1} < \bar{X} < \overline{x_2}$

6. If any number of arithmetic mean be included be as observation in the given set then the arithmetic mean of observation remains the same.

Proof:

Observations: $x_1, x_2, x_3, \ldots \ldots \ldots, x_n \; with \; am = a$

Suppose k number of observations be included in the given set of observations then the new set becomes as

Observations: $x_1, x_2, x_3, \ldots \ldots \ldots, x_n, a, a, \ldots \ldots, a$

Then the combined arithmetic mean is $= \frac{\sum_{i=1}^{n} x_i + ka}{n+k} = \frac{na + ka}{n+k} = a$

7. Sum of all the differences of arithmetic mean from all the observations is zero.

Proof:

To Show

$$\sum_{i=1}^{n} (x_i - \bar{x}) = 0$$

LHS $= \sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$

$$= n\bar{x} - n\bar{x} = 0 = RHS$$

Prepared by

Sanjay Bhattacharya