ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL - 1

Subject : PHYSICS

Date : 7.5.20

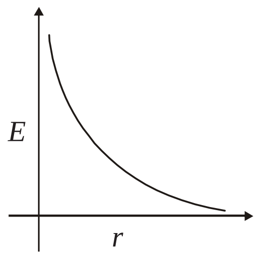
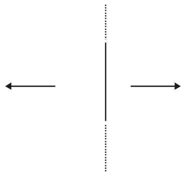
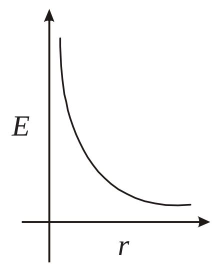
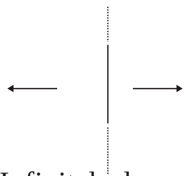
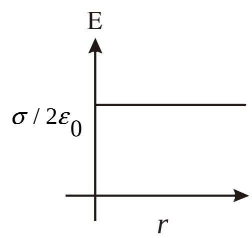
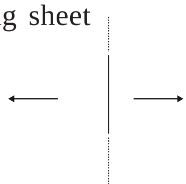
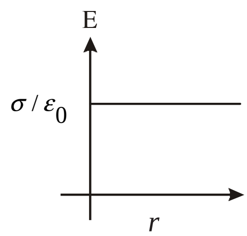
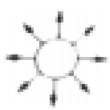
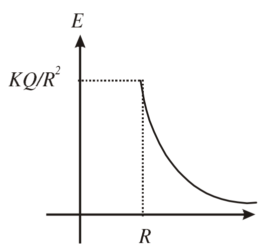
CLASS : XII

Topic : Coloumb's law, electric intensity, Gauss's theorem, related applications

Chapter : Electrostatics

A : Important formulae concepts, diagrams and explanation :

Electric field intensities due to various charge distribution are given in the Table :

Name/Type	Formula	Note	Graph
1. Point Charge 	$\frac{Kq}{\lvert\vec{r}\rvert^2}.\hat{r} = \frac{Kq}{r^3}\vec{r}$	● q is source charge ● $\vec{r}$ is vector drawn from source charge to the test point ● Electric field is non-uniform	
2. Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}.\hat{r} = \frac{2K\lambda\hat{r}}{r}$	● λ is linear charge density (assumed uniform) ● r is perpendicular distance of point from line charge ● $\vec{r}$ radial unit vector drawn from the charge to test point	
3. Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	● σ is surface charge density (assumed uniform) ● $\hat{n}$ is unit normal vector. Electric field intensity is independent of distance.	
4. Infinitely large-charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	● σ is the surface charge density (assumed uniform) ● $\hat{n}$ is the unit vector perpendicular is the surface. ● Electric field intensity is independant of distance.	
5. Uniformly charged hollow conducting/ non-conducting/solid conducting sphere 	(A) for $r \geq R$ $\vec{E} = \frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (B) for $r > R$ $\vec{E} = 0$	● R is radius of the sphere. ● $\vec{r}$ is vector drawn from centre of sphere to the point. ● Sphere acts like a point charge. Placed at centre for points outside the sphere. ● $\vec{E}$ is always along radial direction. ● Q is total charge $(= \sigma 4\pi R^2)$ (σ = surface charge density)	

B : Solved numerical Problems

1. For charges q_1 and q_2, if force between them for some separation in air is F, then force between them in a medium of permitivity ε will be

   Ans. Force in air.

   i.e., $$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

   and force in medium, i.e., $$F_m = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2} \qquad \left(\because \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r\right)$$

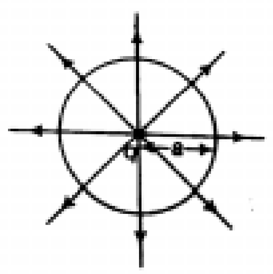
   $$= \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

   $$= \frac{\varepsilon_0}{\varepsilon}\cdot\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

   $$F_m = \frac{\varepsilon_0}{\varepsilon}F$$

2. The electric field in a certain region is acting radially outward and is give by $E = Ar$. A charge contained in a sphere of radius a centred at the origin of the field will be given by

   Ans. $$E = Ar \qquad (1)$$



   Here, $r = a \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}$

   From Eq. (i), we get

   $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2} = Aa \Rightarrow q = 4\pi\varepsilon_0 Aa^3$$

3. When an electric dipole **p** is placed in a uniform electric field **E,** then at what angle between **p** and **E** the value of torque will be maximum?

   Ans. Torque, $\tau = pE\ \sin\theta\hat{n}$

   $$|\tau| = pE\sin\theta$$

   $\therefore$ Torque is maximum, when $\theta = 90^o$

4. Total electric flux coming out of a unit positive charge put in air is —

   Ans. By Gauss's law, ϕ = electric flux through closed surface area

   $$\frac{q_{enclosed}}{\varepsilon_0} \text{ if } q_{enclosed} = 1 \text{ unit}$$

   $$\phi = \frac{1}{\varepsilon_0} = \varepsilon_0^{-1}$$

C : Solution of previous years questions

1. State Gauss' theorem. With the help of this theorem, find out the electrical intensity at any nearby point due to a uniformly charged thin and long straight wire. (2019)

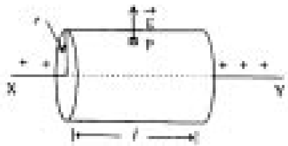
   Ans. : Statement : The net electric flux linked with a closed surface is $\frac{1}{\in}$ times the net charge within the surface.

   Mathematically, $\phi = \oint_S \vec{E}.d\vec{S} = \frac{q}{\in}$ .... (1)

   Here q is the total charge enclosed by the surface and $\in$ is the permittivity of the medium.

   For vacuum, $\phi = \oint_S \vec{E}.d\vec{S} = \frac{q}{\in_0}$

   We consider a then infinitely long straight rod with a uniform linear charge density λ Cm^{-1} placed along XY. We require to calculate electric field at a distance r from the thin rod.



   $\vec{E}$ is a constant over the cylindrical surface. The electric flux only crosses through the curved surface of the cylinder.

   $$\therefore \int_s \vec{E}.\vec{ds} = E \times 2\pi rl$$

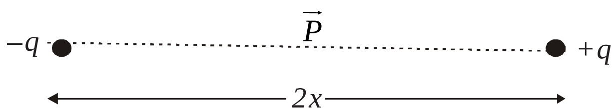
   Hence by Gauss's theorem, $E \times 2\pi rl = \frac{q}{\in_0}$ where $q = \lambda l$

   $$\therefore E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{2\lambda}{\mathrm{r}}\right)$$

   It gives the magnitude of the electric field $\vec{E}$ at a distance r from the line charge.

2. Define electrical dipole moment. An electrical dipole is placed with uniform electric ffield (E) and is rotated to an angle $\angle 0$ = 180°. Find the work done. (2019)

   Ans. Two equal and opposite charges kept at a small separation from an electric dipole.



   Dipole moment : Dipole moment $\vec{P}$ is a measure of the strength of electric dipole.

   It is a vector. The directionof $\vec{P}$ is from negative charge to positive charge.

   Its measurement is $|\vec{P}|$ = magnitude of charge x separation between the two charges.

   = q x $2x$ = $2xq$

   SI unit of dipole moment is (C-m).

   Total work done for producing a deflection θ is

   $\therefore$ W = PE(1-cos)

   Here, $\vec{P}$ = 180°. $\therefore$ W = PE(1 + 1) = 2PE

3. $+q$ point charge is placed at the centre of a hemispherical surface amount of electrical flux crossing through the surface will be

   Ans. Given surface is not closed surface. Hence, we cannot apply the direct result of Gauss's theorem.

   If we draw a complete sphere, then ϕ through complete sphere = $\frac{q}{\varepsilon_0}$

   $\therefore \phi$ through hemisphere = $\frac{1}{2}\left(\frac{q}{\varepsilon_0}\right)$

4. Two point charges separated by a distance d apart each other with a repulsion force 9N. If the separation between them becomes 3d, the force of repulsion will be

   Ans. : $F \propto \frac{1}{d^2} \Rightarrow \frac{F_2}{F_1} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{d_1}{3d_1}\right)^2 = \frac{1}{9}$

   by the problem, $\frac{F_2}{9} = \frac{1}{9}$; $\therefore F_2 = 1N$

D : Short Notes

1. **Coulomb's Law :** If two point charges q_1 and q_2 are distance r apart then force between two charges.

$$F \propto q_1 q_2$$

and $$F \propto \frac{1}{r^2}$$

i.e., $|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ (in free space where ε_0 is permittivity of free space.

$|F| = \frac{q_1 q_2}{4\pi\varepsilon_0 \varepsilon_r r^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 k r^2}$ (in a medium) where

$\varepsilon_r = k = \frac{\varepsilon_m}{\varepsilon_0}$ is relative permittivity of the medium or dielectric constant of the medium.

Vector form of Coulomb's law

$$\vec{F} = \frac{q_1 q_2 \vec{r}}{4\pi\varepsilon_0 r^3}$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\, Nm^2C^{-2}$$

$$\varepsilon_0 = 8.85\times10^{-12}\, C^2N^{-1}m^{-2}$$

Note that ε_r is dimensionless.

Coulomb's law is valid if (i) charges are point charges or spherical charges (ii) distance r between the two charges $\geq 10^{-15}$ m.

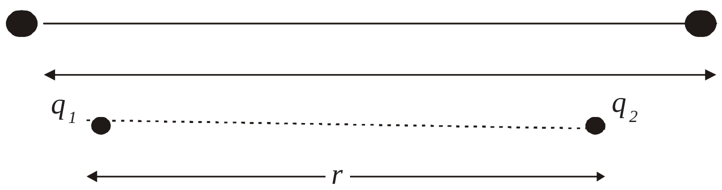


Fig. Coulomb force

2. **Electric Field Intensity** or **Electric Field Strength** is the force experienced by a unit positive charge at that point when placed in an electric field of the given charge. Its unit is N/C or Vm^{-1}.

$$|E| = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{|F|}{q}$$

3. **Electric Flux :** The lines offorce passing through a given area in an electric field is called electric flux. $\phi_E = \int \vec{E}\varepsilon$ If E and S are mutually perpendicular the $\phi_E = 0$. The unit of electric flux is Nm^2C^{-1} and dimensional formula is $[ML^{-3}T^3A^{-1}]$. It is a scalar quantity.

4. **Electric field intensity due to a shell (spherical) having charge *Q* and radius *R***

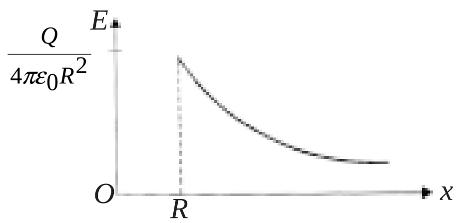


Fig. Electric field due to shell

$$E_{inside} = 0 \qquad x < R$$

$$E_{surface} = \frac{Q}{4\pi\varepsilon_0 R^2} \qquad x = R$$

$$E_{outside} = \frac{Q}{4\pi\varepsilon_0 x^2} \qquad x > R$$

5. **Electric field due to a finite line charge on perpendicular bisector**

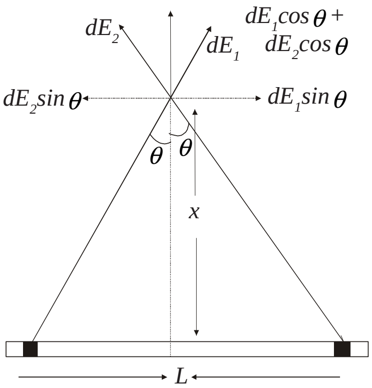


Fig. Electric field due to a line charge along equatorial line

$$E = \frac{Q}{2\pi\varepsilon_0 x\sqrt{L^2+4a^2}}$$

6. **Dipole Moment :** $\vec{P} = q(2\vec{l})$. The direction of electric dipole moment $\vec{p}$ is from negative towards positive charge as shown in the Fig.

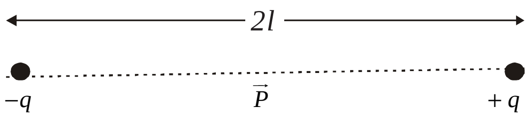


Fig. : Dipole

Electric Field intensity due to a dipole

(a) **Along axial line**

$$E_{axial} = \frac{2px}{4\pi\varepsilon_0(x^2-l^2)^2} \quad \text{(See Fig. below)}$$

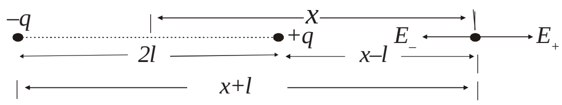


Fig. : Electric field due to a dipole along axial line

for a short dipole $x>>l$

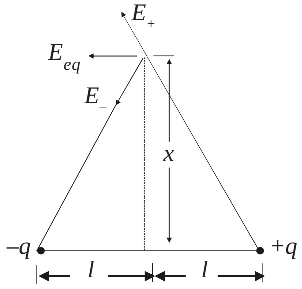
$$E_{axial} = \frac{2p}{4\pi\varepsilon_0 x^3}$$

Note the direction of electric field is parallel to to electric dipole moment.

b) **Electric field along equatorial line**

$$E_{equatorial} = \frac{p}{4\pi\varepsilon_0 (x^2+l^2)^{3/2}}$$

Note that the direction of electric field is antiparalled to dipole movement as shown in Fig.



Electric field along equational line

$$E_{equatorial} = \frac{p}{4\pi\varepsilon_0 x^3}$$ due to a short dipole.

7. **Gauss's Law :**

Gauss's Law is used as an alternative to Coulomb's Law Electric flux $\oint E = \oint \vec{E}.\vec{ds}$. Note that Electric flux does depend on the radius R of the sphere. It only depends on the charge q enclosed in the sphere. According to Gauss's law the closed integral of electric field intensity is equal to $\frac{q}{\varepsilon_0}$ where q is charge enclosed in the closed surface. In other words, total flux through a closed surface enclosed charge q is given by $\oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$.

8. **Electric field due to a long thread (line charge)** having linear charge density λ is $E = \frac{\lambda}{2\pi\varepsilon_0 y} = \frac{18\times10^9 \lambda}{y}$

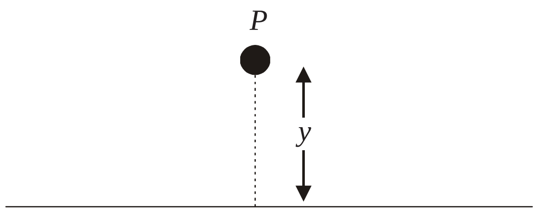


Fig. Electric field due to a long line charge

9. **Electric field due to a sphere charged uniformly with charge Q**

$$E_{inside} = \frac{Qx}{4\pi\varepsilon_0 R^3} \text{ for } x < R$$

$$E_{surface} = \frac{Q}{4\pi\varepsilon_0 R^2} \text{ for } x = R$$

$$E_{outside} = \frac{Q}{4\pi\varepsilon_0 x^2} \text{ for } x > R$$

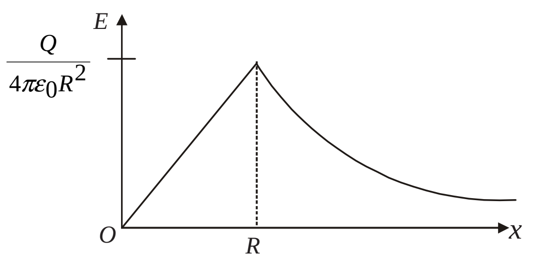


Fig. Electric field due to a sphere charged uniformly

10. Electric Field due to a thin plane sheet (long) of charge density σ $E = \frac{\sigma}{2\varepsilon_0}$.

Electric field due to a charged surface having surface charge density σ $E = \frac{\sigma}{\varepsilon_0}$

Electric field due to a conducting plate $E = \frac{\sigma}{2\varepsilon_0}$.

Electric field between two oppositely charged sheets at any point is $E_{in} = \frac{\sigma}{\varepsilon_0}(= E_1 + E_2)$. Assuming equal surface charge density (for example in a capacitor) $E = \frac{\sigma}{\varepsilon_0}$. Electric field intensity is zero at any point outside the plates as $E_{net} = E_1 - E_2 = 0$, as shown in Fig.

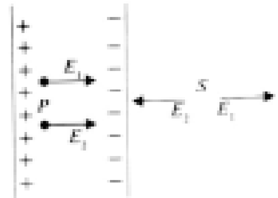


Fig. Electric field due to charged plates

E. Exercise Problems :

1. A pith ball having charge –20 esu and mass 0.1 g remains suspnded at rest in space (in air) 2 cm below an insulated charged sphere. What is the amount of charge on the sphere and what is the nature of tthat charge? [Given, g = 980 cm. s^{-2}]

Solution : Suppose, the charge on the sphere = q

Since the pith ball is at rest,

weight of the pith ball = upward attractive force on the pith ball

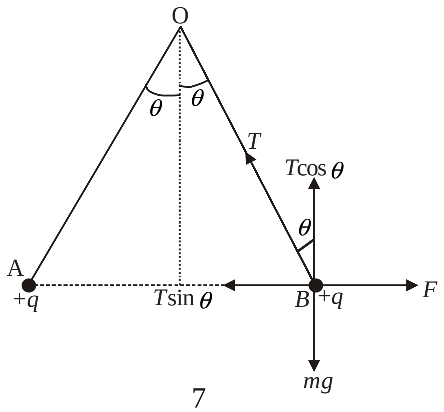
or, $0.1 \times 980 = \frac{20 \times q}{(2)^2}$ or, $q = 19.6$

Since the pith ball is at rest in space, the charge on the sphere is of opposite nature to that of the pith ball. The nature of charge on the pith ball is negative. So the charge on the sphere is positive.

$\therefore$ Charge on the sphere = +19.6 esu.

2. Two similar balls are suspended from a point by two silk threads, each of length l. Each ball of mass m contains q amount of charge. If the angle between the two threads is very small, show that the distance between the centres of the two balls at equilibrium is $x = \left(\frac{2q^2 l}{mg}\right)^{1/3}$

Solution : Let A and B be the equilibrium positions of the two balls. $OA = OB = l$ and $AB = x$. Suppose,the angle of inclination of the two threads with the vertical = θ



At equilibrium, three forces act on each ball; (1) weight of the ball, mg (2) tension in the string, T and (3) maltual repulsive force between the charged balls, F.

Here, $T\sin\theta = F = \dfrac{q^2}{x^2}$; $T\cos\theta = mg$

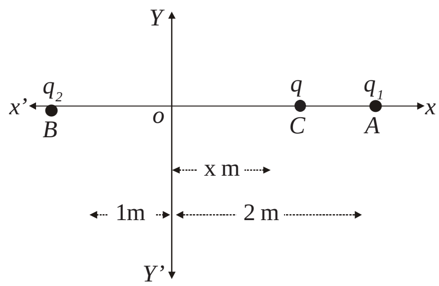
$\therefore$ $\tan\theta = \dfrac{q^2}{mgx^2}$

or, $\sin\theta = \dfrac{q^2}{mgx^2}$ $[\because \theta \text{ is small}, \tan\theta \approx \sin\theta]$

or, $\dfrac{x/2}{l} = \dfrac{q^2}{mgx^2}$ or, $x = \left(\dfrac{2q^2 l}{mg}\right)^{1/3}$

3. Three point charges are lying along the x-axis. If two charges q_1 = –2C and q_2 = –3C are placed at x_1 = 2 m and x_2 = –1 m, respectively and the third positive charge is so located between the first two charges that the resultant force on it is zero, find the position of the third charge.

Solution : In the following Fig., two charges q_1 and q_2 are placed at the points A and B along the x-axis. Suppose, the therd charge q is placed in between q_1 and q_2 at C at a distance x m from the origin O so that the resultant force on it is zero.



$$\therefore \quad \frac{1}{4\pi\epsilon_0}\cdot\frac{qq_1}{(2-x)^2} = \frac{1}{4\pi\epsilon_0}\cdot\frac{qq_2}{(1+x)^2}$$

or, $\dfrac{-2}{(2-x)^2} = \dfrac{-3}{(1+x)^2}$ or, $2(1 + x)^2 = 3(2 - x)^2$

or, $\sqrt{2}(1+x) = \sqrt{3}(2-x)$ or, $(\sqrt{2}+\sqrt{3})x = 2\sqrt{3}-\sqrt{2}$

$$\therefore \quad x = \frac{2\sqrt{3}-\sqrt{2}}{\sqrt{2}+\sqrt{3}} = \frac{(2\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{3-2} = 8+ -3\sqrt{6} \approx 0.65$$

Therefore, the third charge is to be placed at a distance of +0.65 m from the origin O.

4. A negative charge of 20 unit is placed at a distance 50 cm away from a positive charge of 80 unit. Where will the electric field be zero on the line joining the two charges?

Solution : Suppose, +80 unit and –20 unit of charges are placed at A and B respectively. The point where the electric field will be zero cannot lie in between A and B, because in that case intensity could be along the same direction, i.e. along $\overrightarrow{AB}$ for both the charges.

As the charge at A is greater, the point where the resultant intensity is zero will be situated on the right side of the B, say at P.

Suppose, $BP = x$

Electric field at P due to the charge at $B = \frac{20}{x^2}$; along $\overrightarrow{PB}$

Electric field at P due to the charge at $A = \frac{80}{(50+x)^2}$; along $\overrightarrow{AP}$

Since, the resultant intensity at $P = 0$

$\therefore \quad \frac{20}{x^2} = \frac{80}{(50+x)^2}$ or, x = 50, $-\frac{50}{3}$

Now, $x \neq -\frac{50}{3}$ cm, because the point in that case would be situated in between A and B.

So, x = 50 cm; the point where the field is zero at a distance 50 cm from the –20 unit charge on its right side.

5. An electric dipole placed in vacuum is formed by two equal but opposite charges each of magnitude $1\,\mu C$ separated by a distance of 2 cm. Calculate the electric field intensities in the following cases :
   (i) at a point on the axis of the dipole situated at a distance 60 cm away from its centre,
   (ii) at a point on the perpendicular bisector of the dipose situated at a distance 60 cm away from centre.

Solution : Moment of the electric dipole,

$$p = q\,2l = 1 \times 10^{-6} \times 2 \times 10^{-2} = 2 \times 10^{-8}\ C\ m$$

(i) Electric field intensity on the axis of the dipole,

$$E_l = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3} = \frac{9\times10^9\times2\times2\times10^{-8}}{(0.6)^3}$$

= 1666.6 N C^{-1}; along the axis

(ii) Electric field intensity on the perpendicular bisector of the dipole,

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} = \frac{1}{2} E_1$$

= 833.3 N C^{-1}; parallel to axix of the dipole.

Ambarnath Banerjee