ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-7

SUBJECT – MATHEMATICS

Pre-test

Chapter: Relations & Functions **Class: XII**

Topic: Relations **Date: 08.06.2020**

UNIT-I : RELATIONS AND FUNCTIONS (WBCHSE SYLLABUS).

1. Relations and Functions :

Types of relations : reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions :

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

Total Marks : 08

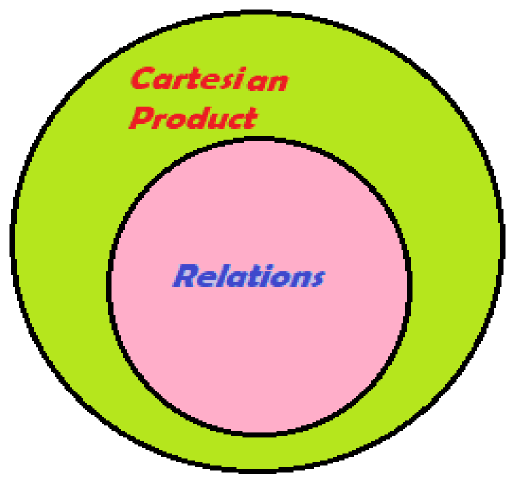
RELATIONS

1. **Definition of Relation :-** A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B.
   The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.
   The second element is called the image of the first element.

Thus , R ⊆ A × B . i.e.

$$(x, y) \in R \Rightarrow (x, y) \in A \times B$$

- If $(x, y) \in R$, then we say x is in relation R to y and write $x \; R \; y$.
- If $(x, y) \notin R$, then we say x is not in relation R to y.



2. **Relation on a set :-** A relation R from a non-empty set A to itself i.e. a subset of A × A is called a relation on set A .

3. **Domain , Range & Co-domain of a relation :-**

   A) Domain :- The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

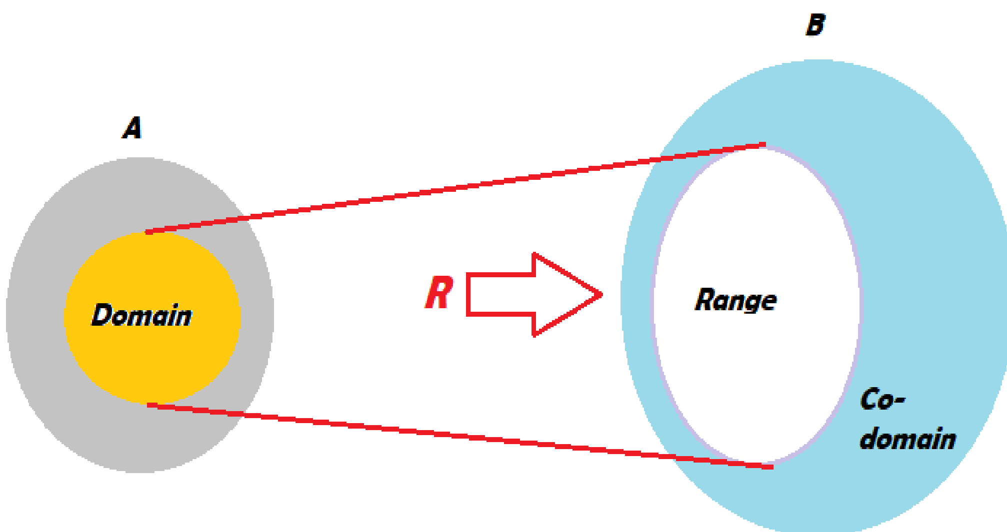
   i.e. $\text{Dom.}(R) = \{x : (x, y) \in R\}$

B) Range : - The set of all second elements of the ordered pairs in a relation R from a set A to a set B is called the range of the relation R.

i.e. $\text{Range}(R) = \{y : (x, y) \in R\}$

C) Co-domain :- The whole set B in a relation R from a set A to a set B is called the co-domain of the relation R.

- We can easily see that , Range(R) ⊆ Co-domain(R) .



4. Types of relations :-

A) Reflexive relation – Let, A be a non-empty set. A relation R on A is reflexive relation if every element of A is related to itself.

i.e. **R is reflexive if (a, a) ∈ R , ∀ a ∈ A.**

B) Symmetric relation – Let, A be a non-empty set. A relation R on A is symmetric relation if $(a,b) \in R \Rightarrow (b,a) \in R$, $\forall\ a , b \ \in A$.

i.e. **R is symmetric if a R b ⇒ b R a ; ∀ a, b ∈ A.**

C) Transitive relation – Let, A be a non-empty set. A relation R on A is transitive relation if $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$, $\forall\ a , b , c \ \in A$.

i.e. **R is symmetric if a R b & b R c ⇒ a R c ; ∀ a, b, c ∈ A.**

D) Equivalence relation – Let, A be a non-empty set. A relation R on A is equivalence relation if R is Reflexive, Symmetric and Transitive.

5. Some important theorems on equivalence relation :-

i. **The union of two equivalence relations on a set is not necessarily an equivalence relation on that set.**

- ✓ Take A={1, 2, 3} and two relations R and S on A defined as follows : R = {(1,1) , (2,2) , (3,3) , (1,2) , (2,1)}
  and S = {(1,1) , (2,2) , (3,3) , (2,3) , (3,2)}

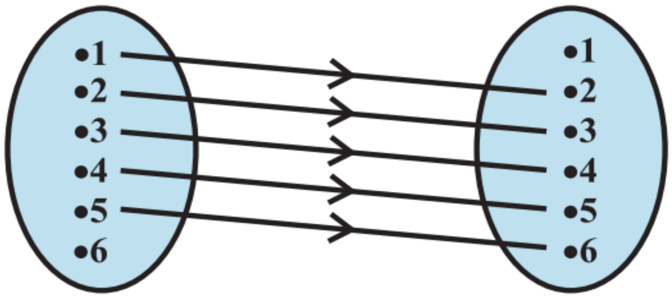
ii. **The intersection of two equivalence relations on a set is an equivalence relation on that set.**

iii. **If R is an equivalence relation on a set A , then the inverse relation R^{-1} is also an equivalence relation on that set.**

6. Some problems on relations : -

❖ **Example 1 :** Let A = {1, 2, 3, 4, 5, 6}. Define a relation R from A to A by R = {(x, y) : y = x + 1 }. Write down the domain, codomain and range of R.

Solution : By the definition of the relation, R = {(1,2), (2,3), (3,4), (4,5), (5,6)}. We can see that the domain ={1, 2, 3, 4, 5,} Similarly, the range = {2, 3, 4, 5, 6} and the codomain = {1, 2, 3, 4, 5, 6}.



❖ **Example 2 :** If R is a relation on ℕ defined as R= $\{(x, y) : x + 3y = 12 \; and \; x, y \in \mathbb{N}\}$. Find the domain and range of R . [HS 2017]

Solution : R = {(3,3) , (6,2) , (9,1)} . Hence , Dom(R)={3,6,9} and Range(R)={3,2,1}.

❖ **Example 3 :** If R is a relation on ℕ defined as R= $\{(x, y) : 2x + y = 41 \; and \; x, y \in \mathbb{N}\}$. Show that R is neither reflexive nor transitive nor symmetric. [HS 2018]

Solution : 3 ∈ ℕ but, (3,3) ∉ ℕ. Hence , not reflexive.
(20 , 1) ∈ R but (1 , 20) ∉ R. Hence , not symmetric.
(12 , 17) ∈ R & (17 , 7) ∈ R but, (12 , 7) ∉ R. Hence not transitive.

❖ **Example 4 :** If R is a relation on ℤ defined as R= $\{(x, y) \in R \Rightarrow (x - y) is \; divisible \; by \; n\}$. Prove that R is an equivalence relation on ℤ.

Solution :

i. $\forall a \in \mathbb{Z}, \; a - a = 0 \; and \; 0 \; is \; divisible \; by \; n \; as \; 0 = 0 \times n. Hence, (a, a) \in R, \forall a \in \mathbb{Z}$.
Hence, R is reflexive on ℤ.

ii. Let, $(a, b) \in R$. $\therefore (a - b) is \; divisible \; by \; n$.
i.e. $(a - b) = nk, k \in \mathbb{Z}. \Rightarrow \; (b - a) = -nk = n(-k). Now, as \; k \in \mathbb{Z}, (-k) \in \mathbb{Z}$
$\therefore (b - a) is \; also \; divisible \; by \; n \; \Rightarrow (b, a) \in R$. Hence, R is symmetric on ℤ.

iii. Let, $(a, b) \in R$ & $(b, c) \in R$. $\therefore (a - b) is \; divisible \; by \; n$ and $(b - c) is \; divisible \; by \; n$.
$\therefore (a - b) + (b - c) = (a - c) is \; divisible \; by \; n$. {As, a, b, c are in ℤ }.
$\Rightarrow (a, c) \in R$. Hence, R is transitive on ℤ.

Hence, R is an equivalence relation on ℤ. (proved)

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