ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

FOR GOD AND COUNTRY

STUDY MATERIAL-22

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Integration **Class: XII**

Topic: Some standard Forms **Date: 03.07.2020**

-:Some standard Forms:-

(Part 1)

Integrals of the form:

1. $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \sqrt{ax^2+bx+c}\, dx$

Convert $ax^2+bx+c = a\left\{\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}\right\}$

And then use formulas

(a) $\int \frac{dx}{x^2-a^2}$, $\int \frac{dx}{x^2+a^2}$, $\int \frac{dx}{a^2-x^2}$ for $\int \frac{dx}{ax^2+bx+c}$

(b) $\int \frac{dx}{\sqrt{x^2-a^2}}$, $\int \frac{dx}{\sqrt{x^2+a^2}}$, $\int \frac{dx}{\sqrt{a^2-x^2}}$ for $\int \frac{dx}{\sqrt{ax^2+bx+c}}$

(c) $\int \sqrt{x^2-a^2}\, dx$, $\int \sqrt{x^2+a^2}\, dx$, $\int \sqrt{a^2-x^2}\, dx$ for

$\int \sqrt{ax^2+bx+c}\, dx$.

Solved Examples :-

Example 1. Evaluate $\int \frac{1}{x^2+x+1}dx.$

Solution:

$$I=\int \frac{1}{x^2+x+1}dx=\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$$

Example 2. Evaluate $\int \frac{1}{\sqrt{2x^2+3x+2}}dx.$

Solution:

$$I=\int \frac{1}{\sqrt{2x^2+3x+2}}dx=\frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}}dx$$

$$=\frac{1}{\sqrt{2}}\ln\left(\left(x+\frac{3}{4}\right)+\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}\right)+c$$

Example 3. Evaluate $\int \sqrt{x^2 - x}\, dx$.

Solution:

$$I = \int \sqrt{x^2 - x}\, dx = \int \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\, dx$$

$$I = \frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x} - \frac{1}{8}\ln\left(\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x}\right) + c$$

Example 4. Evaluate $\int \frac{1}{2x^2 + x + 1} dx.$

Solution:

$$I = \int \frac{1}{2x^2 + x + 1} dx = \frac{1}{2}\int \frac{1}{\left(x + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} dx$$

$$I = \frac{1}{2} \cdot \frac{4}{\sqrt{7}} \tan^{-1}\left(\frac{\left(x + \frac{1}{4}\right)}{\left(\frac{\sqrt{7}}{4}\right)}\right) + c$$

$$I = \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{(4x + 1)}{\sqrt{7}}\right) + c$$

Example 5. Evaluate $\int \frac{1}{\sqrt{2-3x-x^2}}dx.$

Solution:

$$I=\int \frac{1}{\sqrt{2-3x-x^2}}dx=\int \frac{1}{\sqrt{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x+\frac{3}{2}\right)^2}}dx$$

Put $\left(x+\frac{3}{2}\right)=t$. Then $dx=dt$.

$$I=\int \frac{1}{\sqrt{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}}dt=\sin^{-1}\left(\frac{2t}{\sqrt{17}}\right)+c=\sin^{-1}\left(\frac{2x+3}{\sqrt{17}}\right)+c$$

Example 6. Evaluate $\int \sqrt{2ax-x^2}\,dx.$

Solution:

$$I=\int \sqrt{2ax-x^2}\,dx=\int \sqrt{a^2-a^2+2ax-x^2}\,dx$$

$$=\int \sqrt{a^2-(a^2-2ax+x^2)}\,dx=\int \sqrt{a^2-(x-a)^2}\,dx$$

Put $(x-a)=t$. Then $dx=dt$.

$$I=\int\sqrt{a^2-t^2}\,dt=\frac{t}{2}\sqrt{a^2-t^2}+\frac{a^2}{2}\sin^{-1}\frac{t}{a}+c$$

$$I=\frac{(x-a)}{2}\sqrt{a^2-(x-a)^2}+\frac{a^2}{2}\sin^{-1}\frac{(x-a)}{a}+c$$

$$I=\frac{(x-a)}{2}\sqrt{2ax-x^2}+\frac{a^2}{2}\sin^{-1}\frac{(x-a)}{a}+c$$

Example 7. Evaluate $\int\sqrt{x^2+8x+12}\,dx$.

Solution:

$$I=\int\sqrt{x^2+8x+12}\,dx=\int\sqrt{(x+4)^2-(2)^2}\,dx$$

Put $(x+4)=t$. Then $dx=dt$.

$$I=\int\sqrt{t^2-2^2}\,dt=\frac{t}{2}\sqrt{t^2-2^2}-\frac{4}{2}\ln\left|t+\sqrt{t^2-2^2}\right|+c$$

$$I=\frac{(x+4)}{2}\sqrt{(x+4)^2-2^2}-\frac{4}{2}\ln\left|(x+4)+\sqrt{(x+4)^2-2^2}\right|+c$$

$$I=\frac{(x+4)}{2}\sqrt{x^2+8x+12}-2\ln\left|(x+4)+\sqrt{x^2+8x+12}\right|+c$$

Example 8. Evaluate $\int \frac{1}{\sqrt{x^2-4x+2}}dx.$

Solution:

$$I=\int \frac{1}{\sqrt{x^2-4x+2}}dx=\int \frac{1}{\sqrt{(x-2)^2-(\sqrt{2})^2}}dx$$

Put $(x-2)=t$. Then $dx=dt$.

$$I=\int \frac{1}{\sqrt{t^2-(\sqrt{2})^2}}dt$$

$$\Rightarrow I=\ln\left|t+\sqrt{t^2-(\sqrt{2})^2}\right|+c$$

$$\Rightarrow I=\ln\left|(x-2)+\sqrt{x^2-4x+2}\right|+c$$

- **Prepared by**

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