ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-21

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Integration **Class: XII**

Topic: Integration By Parts **Date: 02.07.2020**

–: Integration By Parts :–

Some standard results:-

1. $\int e^x (f(x)+f'(x))dx = e^x f(x)+c$

2. $\int (f(x)+xf'(x))dx = xf(x)+c$

3. $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(-b\cos bx + a\sin bx)+c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}} \sin\left(bx - \tan^{-1}\frac{b}{a}\right)+c$$

4. $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(b\sin bx + a\cos bx)+c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}} \cos\left(bx - \tan^{-1}\frac{b}{a}\right)+c$$

5. $\int \sqrt{a^2-x^2}\ dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$

6. $\int \sqrt{x^2-a^2}\ dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|+c$

7. $\int \sqrt{x^2+a^2}\ dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|+c$

Solved Examples :-

Example 1. Evaluate $\int e^x(1-\cot x+\cot^2 x)\,dx.$

Solution:

$$\int e^x(1-\cot x+\cot^2 x)dx=\int e^x(-\cot x+\operatorname{cosec}^2 x)dx=-e^x\cot x+c$$

Example 2. Evaluate $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx.$

Solution:

$$\left(\frac{1-\sin x}{1-\cos x}\right)=\frac{1}{2\sin^2\frac{x}{2}}-\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}=\operatorname{cosec}^2\frac{x}{2}-\cot\frac{x}{2}$$

$$\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\int e^x\left(\operatorname{cosec}^2\frac{x}{2}-\cot\frac{x}{2}\right)dx=-e^x\cot\frac{x}{2}+c$$

Example 3. Evaluate $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx.$

Solution:

$$I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx\Rightarrow I=\frac{e^x}{x}+c$$

Example 4. Evaluate $\int (x\cos x + \sin x)dx.$

Solution:

$$I = \int (x\cos x + \sin x)dx = x\sin x + c$$

Example 5. Evaluate $\int \left(\frac{x+\sin x}{1+\cos x}\right)dx.$

Solution:

$$I = \int \left(\frac{x+\sin x}{1+\cos x}\right)dx = \frac{1}{2}\int \left(x\sec^2\frac{x}{2}\right)dx + \int \tan\frac{x}{2}dx$$

$$I = \frac{1}{2}\frac{x\tan\frac{x}{2}}{\frac{1}{2}} - \int \tan\frac{x}{2}dx + \int \tan\frac{x}{2}dx + c = x\tan\frac{x}{2} + c$$

Example 6. Evaluate $\int \sqrt{x^2+a^2}\,dx.$

Solution:

$$I = \int \sqrt{x^2+a^2}dx = \sqrt{x^2+a^2}\int 1dx - \int \frac{2x^2}{2\sqrt{x^2+a^2}}dx$$

$$= x\sqrt{x^2+a^2} - \int \frac{x^2+a^2}{\sqrt{x^2+a^2}}dx + \int \frac{a^2}{\sqrt{x^2+a^2}}dx$$

$$\Rightarrow I = x\sqrt{x^2+a^2} - I + a^2 \ln\left|x+\sqrt{x^2+a^2}\right| + c$$

$$\Rightarrow 2I = x\sqrt{x^2+a^2} + a^2 \ln\left|x+\sqrt{x^2+a^2}\right| + c$$

$$\Rightarrow I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \ln\left|x+\sqrt{x^2+a^2}\right| + c$$

Example 7. Evaluate $\int e^x \sin x\, dx$.

Solution:

$$\int e^x \sin x\, dx = \frac{e^x}{1^2+1^2}(1\cdot \sin x - 1\cdot \cos x) + c = \frac{e^x}{2}(\sin x - \cos x) + c$$

- **Prepared by**

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