ST. LAWRENCE HIGH SCHOOL

A JESUIT CHRISTIAN MINORITY INSTITUTION

STUDY MATERIAL-20

SUBJECT – MATHEMATICS

Pre-Test

Chapter: Integration **Class: XII**

Topic: Integration By Parts **Date: 01.07.2020**

–: Integration By Parts :–

The working rule:-

If F and G are two functions of x, then integral of the product of these two functions is given by

$$\int F \cdot G\, dx = F\int G\, dx - \int\left(\frac{dF}{dx}\int G\, dx\right)dx$$

Or we can say that the integral of the product of two functions = (First function) × (Integral of second function) – Integral of {(Differentiation of first function) × (Integral of second function)}.

✓ **The easily integrable function in most cases is chosen as the second function.**

✓ **However we can follow the "LIATE" rule to choose the 1st and the 2nd function. The function which comes first in the following list could be taken as the first function.**

- **L – Logarithmic functions.**
- **I – Inverse trigonometric functions.**
- **A – Algebraic functions.**
- **T – Trigonometric functions.**
- **E – Exponential functions.**

Solved Examples :-

Example 1. Evaluate $\int x \cdot \sin x \, dx$.

Solution:

$$I = \int x \cdot \sin x \, dx = -x \cos x + \int \cos x \, dx = -x \cos x + \sin x + c$$

Example 2. Evaluate $\int x \sec^2 x \, dx$.

Solution:

$$I = \int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx = x \tan x + \ln \cos x + c$$

Example 3. Evaluate $\int x^3 \ln x \; dx$.

Solution:

$$I = \int x^3 \ln x \; dx = \frac{x^4}{4} \cdot \ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

$$= \frac{x^4}{4} \cdot \ln x - \frac{x^4}{16} + c$$

Example 4. Evaluate $\int (f(x)g''(x) - g(x)f''(x))dx.$

Solution:

$$I = \int [(f(x)g''(x) - g(x)f''(x))]\, dx = \int f(x)g''(x)dx - \int g(x)f''(x)dx$$

$$I = [f(x)g'(x) - \int f'(x)g'(x)dx] - [g(x)f'(x) - \int f'(x)g'(x)dx]$$

$$= f(x)g'(x) - g(x)f'(x)$$

Example 5. Evaluate $\int \sec^3\theta\, d\theta.$

Solution:

$$I = \int \sec^3\theta\, d\theta = \sec\theta \int \sec^2\theta\, d\theta - \int \tan\theta(\sec\theta\tan\theta)\, d\theta$$

$$= \sec\theta \cdot \tan\theta - \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= \sec\theta \cdot \tan\theta - \int \sec^3\theta\, d\theta + \int \sec\theta\, d\theta$$

$$\Rightarrow I = \sec\theta \cdot \tan\theta - I + \int \sec\theta\, d\theta$$

$$\Rightarrow I = \frac{1}{2}[\sec\theta \cdot \tan\theta] + \frac{1}{2}\ln|\sec\theta + \tan\theta| + c$$

Example 6. Evaluate $\int \frac{x^2 dx}{(x\sin x+\cos x)^2}$.

Solution:

$$I=\int \frac{x^2}{(x\sin x+\cos x)^2}\,dx=\int \frac{x\cos x}{(x\sin x+\cos x)^2}\cdot\frac{x}{\cos x}\,dx$$

$$I=\frac{-1}{(x\sin x+\cos x)}\cdot\frac{x}{\cos x}+\int \frac{1}{(x\sin x+\cos x)}\cdot\frac{\cos x+x\sin x}{\cos^2 x}\,dx$$

$$I=\frac{-1}{(x\sin x+\cos x)}\cdot\frac{x}{\cos x}+\int \sec^2 x\,dx$$

$$I=\frac{-1}{(x\sin x+\cos x)}\cdot\frac{x}{\cos x}+\tan x+c$$

Example 7. Evaluate $\int \sqrt{x^2+a^2}\,dx$.

Solution:

$$I=\int \sqrt{x^2+a^2}\,dx=\sqrt{x^2+a^2}\int 1dx-\int \frac{2x^2}{2\sqrt{x^2+a^2}}dx$$

$$=x\sqrt{x^2+a^2}-\int \frac{x^2+a^2}{\sqrt{x^2+a^2}}dx+\int \frac{a^2}{\sqrt{x^2+a^2}}dx$$

$$\Rightarrow I=x\sqrt{x^2+a^2}-I+a^2\ln\left|x+\sqrt{x^2+a^2}\right|+c$$

$$\Rightarrow 2I=x\sqrt{x^2+a^2}+a^2\ln\left|x+\sqrt{x^2+a^2}\right|+c$$

$$\Rightarrow I=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right|+c$$

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